THE DETERMINATION OF AQUIFER CONSTANTS USING WATER - LEVEL OBSERVATIONS IN THE PUMPING WELL

By

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I. INTRODUCTION

Public water-supply systems in the State of Florida served in 1960 3,320,000 inhabitants with an average daily consumption of 530 millions of gallons. Of that total population 98.6 per cent were served with water secured from ground-water sources. Of the average daily consumption 94.3 per cent was obtained from ground-water supplies. Ground water was also used in Florida to supply 100 per cent of the domestic rural water consumption with an average of 86 millions of gallons per day. (MacKichan and Kammerer, 1961, pp. 13-15). In total 214 billions of gallons of water were withdrawn in 1960 from ground water bearing formations in the State. The above figures indicate the significance of ground-water supplies and resources in the State of Florida.

C. S. Conover (1961, pp. 1-2) has stated:

> "The need for broader appreciation, evaluation and management of our ground-water resources is becoming more evident every day.
>
> " . . . The flow in streams is such that water from a large area can be gaged at a single location. Surface reservoirs also can be gaged at a single location.
>
> " . . . Though stream channels are not simple conduits, they can be easily mapped and measured. The same is not true with aquifers. . . . Therefore, aquifers are not easily mapped. However, they must be mapped, their water-bearing characteristics determined, and their hydrologic regimen evaluated before decisions can be made as to optimum development and management of the ground-water resource over a long period of time."

The water-bearing characteristics of an aquifer are defined by the aquifer constants: permeability and storage coefficient. The determination of aquifer constants is, therefore, one of the most important factors in the development of a source of ground-water supply.

Permeability and storage coefficient are best determined by field tests in which the lowering of the water level at any point in the vicinities of a pumping well is correlated with its pumping rate, the time elapsed since pumping began and the distance from that point to the pumping well. The non-equilibrium formula developed by Theis (1935, pp. 519-524) expresses the relation among the lowering of the water level (drawdown) in a water-bearing formation, the time factor, the rate of pumping, and the two aquifer constants in which we are interested: the storage coefficient and the transmissibility. The transmissibility of an aquifer is equal to its horizontal permeability times the thickness of the aquifer.

The development by Theis of his non-equilibrium formula in 1935 was a major advancement in ground-water hydrology. However, the direct numerical solution of the formula was not feasible; therefore, several graphical methods have since been developed for its solution.

J. C. Jaeger (1959, pp. 516-564) proposed a method for the determination of the transmissibility and the storage coefficient based on an easy numerical solution of the non-equilibrium formula. Later, Garcia Bengochea (1960, 83 pp.) evaluated Jaeger's method with field data, and established a procedure to determine both aquifer constants by the use of the numerical solution proposed by Jaeger.

II. SCOPE

The purpose of this paper is to study the possibilities of the determination of aquifer constants by the application of the Jaeger method to water level observations in the pumping well. If that is proven to be possible, aquifer constants could be much more easily calculated than with other existing methods.

The determination of aquifer constants is presently being made by the application of graphical methods developed after Theis non-equilibrium formula. These methods are used with water level data collected from one or more observation or satellite wells located at adequate distances from another well which is pumped at a certain rate. It is therefore necessary to have a pumping well and one or several satellite wells in which careful measurements of the water level are made. If no observation well is available, existing graphical methods do not give reliable results. Only the transmissibility can be safely determined with them by using the recovery data plotted as a residual drawdown curve (Briggs, 1962, p. 6) (Skibitzke, 1958).

The method proposed by Jaeger (1959, pp 561-564) makes use of the ratio of the values of the drawdowns obtained from an observation well during a pumping test at any two times, and a table or graph mathematically computed from the theory of the non-equilibrium formula. This method has been evaluated (Garcia Bengochea, 1960, 83 pp.) with data obtained also in satellite wells from four different pumping tests. Conclusion from that evaluation indicated that the Jaeger method was not subject to the possible inaccuracies of the existing graphical method and also that it showed the reliability of the values obtained.

The applicability of the Jaeger method to determine aquifer constants utilizing water level observations in the pumping well has been evaluated in this paper with results obtained also from four different pumping tests. In the first two tests the method was applied with data collected by the writer from pumping tests performed long before this application was being considered. Results obtained by the use of water level observations in the pumping well were compared with those calculated with observations in the satellite wells. Experience from these first two applications was employed to program data collection for a new pumping test in which water levels were recorded both at the pumping well and at an observation well. Results computed from data collected from both wells were again compared. Finally another pumping test was performed in a locality where no near-by satellite well was available. Results from this last test have been also evaluated.

Pumping tests from which data have been applied and results evaluated in this work are shown in Table I. That table also shows the pumping rate during each test, the distance to the observation well, and the condition of the aquifer.

TABLE I

PUMPING TESTS FROM WHICH DATA HAVE BEEN USED

Location of Test	Pumping Rate gpm	Distance to Satellite Well feet	Condition of Aquifer
Acueducto de Marianao La Habana, Cuba Well No. 13	4,980	984	Water Table
Cia. Cervecera de las Antillas La Habana, Cuba Well No. 2	1,280	502	Water Table
City of Fort Pierce Florida, U.S.A. Test Well No. 10A	400	100	Artesian
City of Leesburg Florida, U.S.A. Well No. 12	1,250	0	Artesian

III. DEFINITIONS OF TERMS

The following definitions should facilitate the understanding of the succeeding chapters.

Aquifer. Ground water occurs in permeable formations known as aquifers, that is, geologic formations having structures that permit appreciable amounts of water to move through them under ordinary field conditions. Ground water reservoir and water-bearing formation are commonly used synonyms (Todd, 1959, p.15).

Aquifer under Artesian Condition. Ground water in an aquifer is under artesian condition if the aquifer is overlain by a confining bed of impervious material and the water level in a tightly cased well penetrating the aquifer rises above the bottom of the confining bed. The surface formed by contouring or connecting the heights of the water level in tightly cased wells tapping the aquifer is an imaginary pressure surface standing above the body of the aquifer and consequently receives the name of piezometric surface (Ferris, 1949, p. 216).

Aquifer under Water-Table Condition. An aquifer is said to be under water-table conditions when the top of the aquifer is a surface exposed to the soil atmosphere. This surface is called "water-table" and although the term "piezometric surface" can be applied also to the water-table surface, the reverse is not true (Ferris, 1949, p. 216).

Aquiclude. In contradistinction to an aquifer, an aquiclude is an impermeable formation which may contain some amount of water but is incapable of transmitting significant water quantities, clay for instance (Todd, 1959, p.15).

Aquifuge. An impermeable formation neither containing nor transmitting water, like solid granite, is called an aquifuge (Todd, 1959, p.15).

Drawdown. When a well is pumped, water is removed from the aquifer surrounding the well and the water table or piezometric surface, depending upon the type of aquifer, is lowered. The drawdown at a given point is the vertical distance the water level is lowered (Todd, 1959, p.81).

Effective Radius. The "Effective Radius of a Well" is defined as that distance, measured radially from the axis of the well, at which the theoretical drawdown based on the logarithmic head distribution equals the actual drawdown just outside the screen or well (Jacob, 1947, p.1048).

Leakage Coefficient. Or "Leakance," may be defined as the rate of flow which crosses a unit area of the interface between the main aquifer and its semiconfining bed, if the difference between the head in the main aquifer and in that one supplying leakage is unity (Hantush, 1956, p.702). It is actually the coefficient of vertical permeability of the semipermeable confining bed divided by its thickness.

Permeability. The permeability of a formation, rock or soil, with respect to water is its capacity for transmitting water under pressure. This capacity can be quantitatively defined as the rate of discharge of water through a unit cross-section area of the formation perpendicular to the direction of flow if the hydraulic gradient is 100 per cent (Meinzer, 1923, p.44).

Permeability is usually expressed, and it has also been defined by Meinzer (Stearns, 1928, p.148), as the rate of flow of water, in gallons per day, through a cross-sectional area of 1 square foot under a hydraulic gradient of 100 per cent at a temperature of 60 degrees Fahrenheit.

Recovery. It is the vertical distance the water level rises in a well or in a formation immediately after and produced by stopping the flow from a near-by well or wells.

Specific Yield. The specific yield of a formation is defined as the quantity of water yielded by gravity drainage from saturated water material, and it is expressed as a percentage of the total volume of the material drained (Ferris, 1949, p.210).

Storage Coefficient. Storage coefficient is defined as the volume of water that an aquifer releases from storage per unit surface area of aquifer per unit change in the component of head normal to that surface (Todd, 1959, p.31). In other words, it is the amount of water, expressed as a decimal fraction of unity, that is released from storage from a vertical column of the aquifer of a unit horizontal cross-section when the piezometric head on that column drops one unit head.

Transmissibility. The term coefficient of transmissibility, or just transmissibility as it is commonly used, was introduced by Theis (1935, p.520) with the development of his non-equilibrium formula. It is defined as the rate of flow of water in gallons per day through a vertical strip of the aquifer 1 foot wide and extending its full saturated height under a hydraulic gradient of 100 per cent (Ferris, 1949, p. 206). Actually the term "at a temperature of 60 degrees Fahrenheit" should be added to the definition as water viscosity changes with temperature, but for practical purposes this last part could be neglected, as the prevailing temperature in the aquifer water is always assumed.

From the comparison of the definitions of permeability and transmissibility we can see that both terms express the capacity of a formation for transmitting water. It is also easily seen that the transmissibility is equal to the permeability multiplied by the full saturated height of the aquifer, assuming of course that both factors are expressed in the same system of units.

IV. FUNDAMENTAL THEORY

Hagen (1839, pp. 423-442) and Poiseuille (1846, p. 433) were the first to study the law of flow through capillary tubes. They found that the rate of flow is proportional to the hydraulic gradient.

Later Darcy (1856) verified this observation and demonstrated its applicability to water percolating through the capillary interstices of filter sands. Darcy expressed this law by means of the formula

$$v = \frac{P\,h}{l}$$

in which

"v" is the velocity of the water through a column of permeable material,

"l" is the length of the column,

"h" is the difference in head at the ends of the column, and

"P" is a constant that depends on the character of the material, especially on the size and arrangements of the grains.

As it is usually more essential to determine the quantity of water flowing through a certain cross-section of permeable material than to determine the velocity through the material, Darcy's law is then expressed as:

$$Q = PIA$$

in which

"Q" is the rate of flow of water discharged,

"P" is the same constant previously defined and also called permeability,

"I" is the hydraulic gradient, and

"A" is the cross-sectional area through which the water flows.

This formula serves as a basis for determining the quantities of ground water that percolate from areas of recharge to areas of discharge, and consequently it is used for determining the safe yield of underground reservoirs (Wenzel, 1942, pp. 3-4).

An important step of progress in ground-water hydrology was made by Theis (1935, pp. 519-524) with the development of the non-equilibrium formula which expresses the relation among drawdown, time, transmissibility, and storage coefficient. The development of the formula was based on the assumption that Darcy's law is analogous to the law of the flow of heat by conduction and thus the mathematical theory of heat conduction is largely applicable to hydraulic theory.

Some years after Theis' development of the non-equilibrium formula, Jacob (1940, pp. 574-586) demonstrated the derivation of that formula using strictly hydraulic concepts.

The non-equilibrium formula is the final equation for the drawdown of the water level in the vicinity of a discharging well, as developed from an equation expressing the changes in temperature due to a type of source, that is analogous to a recharging or discharging well under certain conditions. It is expressed:

$$s = \frac{114.6\ Q}{T} \int_u^{\infty} \frac{e^{-x}}{x}\ dx \qquad (1)$$

in which

"s" is the drawdown in feet at any point in the vicinity of a well discharging at a uniform rate,

"Q" is the rate of flow discharged by a well in gallons per minute,

"T" is the transmissibility of the aquifer in gallons per day per foot,

"u" is equal to $$\frac{1.87\ r^2\ S}{T\ t} \qquad (2)$$

"r" is the distance of the discharge well to the point of observation in feet,

"S" is the storage coefficient of the aquifer expressed as a decimal fraction, and

"t" is the time the well has been pumped in days.

The expression

$$\int_u^{\infty} \frac{e^{-x}}{x}\ dx$$

is not directly integrable, it is symbolically written as W(u), and it is usually called the "well function of u." However, its value can be computed by the following series:

$$W(u) = \int_u^{\infty} \frac{e^{-x}}{x}\ dx =$$

$$0.577216 - \ln.u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} \ldots (3)$$

Values of W(u) for values of u between 10^{-15} and 9.9 are given by published tables (Wenzel, 1942, p. 89) and are also graphically given in Figures 1, 2, and 3.

The non-equilibrium formula is based on the following assumptions (Wenzel, 1942, pp. 87-88):

1. - The water-bearing formation is homogeneous and isotropic,

2. - The formation has an indefinite areal extent,

3. - The discharge well penetrates the entire thickness of the formation,

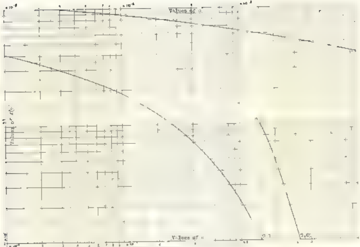
4. - The transmissibility is constant at all places and at all times,

5. The discharge well has an infinitesimal diameter, and

6. - Water taken from storage by the decline in water level is drained instantaneously with the decline in head.

These assumptions are very seldom met in the field, but it is often true that these deviations are small enough so that the non-equilibrium formula can be applied. However, the closer field conditions accomplish these requirements, the better results will be obtained with that formula.

The formula can be used in two ways. If the transmissibility and the storage coefficient are known, the drawdown can be computed for any time and at any point of the aquifer affected by the well. If the drawdowns are known, the transmissibility and the storage coefficient can be computed. If the drawdowns are known, both constants can be computed from the drawdown curve of one well or from the drawdowns that are observed at any one time in a line of wells (Wenzel, 1942, p. 88).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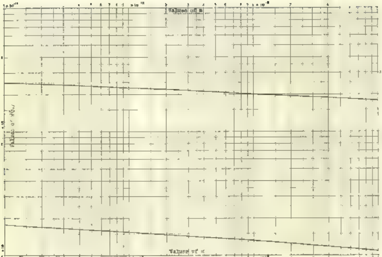


Fi 2.— [illegible] Curve for [illegible] to 1x10^{-5}

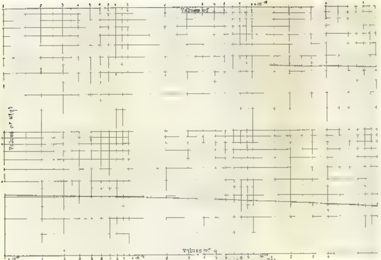


Fig. 2 [illegible] Curves for [illegible] Formula. Values of [illegible]

As the transmissibility T appears twice in the equation, once in the argument of the function and again as a divisor of the exponential integral, the formula cannot be solved directly for T and S when the drawdowns are known. Therefore, investigators have developed several graphical methods for the solution of this formula and are explained in the following chapter.

Two of the most common and significant deviations met under field conditions are: (1) that the aquifer may not have areal extent large enough to be considered as of indefinite extent, and (2) that the aquifer is replenished by vertical leakage through overlying or underlying semipermeable confining beds.

In order to meet the first type of departure, the non-equilibrium formula can be modified for use in locations where the pumped aquifer is adjacent to another formation having a considerably different transmissibility. Stallman (1952, 5 pp.) has developed such modification by an application of the image well theory.

The second departure and the most common in Florida is that of an elastic artesian aquifer which is recharged by vertical leakage from overlying or underlying confining beds. Generally, the confining beds only retard the vertical flow of ground water rather than prevent it.

Leaky-aquifer problems were first analyzed in detailed by Glee (1930) and later on by Steggewents and Van Nes (1939, pp. 561-563), by Jacob (1946, pp. 198-205), by Hantush and Jacob (1954, pp. 917-936; 1955, pp. 95-100) and by Hantush (1956, pp. 702-714). Very recently Cooper (1963) has developed "Type Curves for Nonsteady Radial Flow in an Infinite Leaky Aquifer" by which the transmissibility and storage coefficient of a leaky artesian aquifer and the leakage coefficient of its confining bed may be determined by a graphical method which will be also explained in Chapter VII.

V. SUMMARY OF EXISTING METHODS OF DETERMINING CONSTANTS OF AQUIFERS WITH NO VERTICAL LEAKAGE

We have seen from the previous chapter that the non-equilibrium formula can be used for the determination of the transmissibility and the storage coefficient of an aquifer when the drawdowns in a well at different time intervals or the drawdowns in a line of wells at any specified time are known. We have also seen that, because of the nature of that formula, it cannot be solved directly for T and S when the drawdowns at any point or points and at different time intervals or at any specific time are known.

In order to overcome this difficulty, several graphical methods have been devised by different investigators. These are

Theis method (Wenzel, 1942, pp. 88-89),

Cooper and Jacob method (Cooper and Jacob, 1946, pp. 526-534), also "Straight-Line" or "Semilog-Plot" method and

Chow method (Chow, 1952, pp. 397-404).

In general, these methods can be used with drawdowns obtained in an observation well at different time intervals after pumping is started in a main well, with the recovery of an observation well after pumping has been stopped, and also with the drawdowns in a line of observation wells at any time interval after pumping has been started in the main well.

In actual field practice the first two methods, with drawdowns obtained in an observation well at different time intervals after pumping has been started, are the most commonly used. Therefore, and for the sake of simplification, these procedures are the ones to be used in this work and are explained below.

Theis Method. When the transmissibility and the storage coefficient are to be determined from the drawdowns in an observation well, the logarithm of the drawdown is plotted against the logarithm of the reciprocal of the time since pumping began (s against $\frac{1}{t}$). If the formation was entirely homogeneous and the water was discharged instantaneously with the fall in water level, all points so plotted would fall on a smooth curve. The curve so determined is a segment of the type curve produced by plotting the logarithm of the value of the integral W(u), against the logarithm of the value u. This last plotting is shown in Figures 1, 2, and 3.

The last statement can be easily seen if Equations (1) and (2),

$$s = \frac{114.6\ Q}{T} \int_{u}^{\infty} \frac{e^{-x}}{x}\ dx \qquad (1)$$

$$u = \frac{1.87\ r^2 S}{T\ t} \qquad (2)$$

are arranged in a more convenient form like

$$s = \frac{114.6\ Q}{T}\ W(u) \qquad (1a)$$

$$W(u) = \left[\frac{T}{114.6\ Q}\right] s \qquad (4)$$

$$u = \left[\frac{1.87\ r^2 S}{T}\right] \frac{1}{t} \qquad (5)$$

The brackets enclose all the terms that are constant. The similarity of the form exhibited by Equations (4) and (5) should suggest that, if we ignore the bracketed coefficients, which are constants, the term W(u) will vary with the term u in the same manner as s will vary with $\frac{1}{t}$ (Brown, 1953, pp. 852-853).

Therefore, if the observed drawdown values in the observation well are plotted on transparent paper against $\frac{1}{t}$ to a logarithmic scale the same as that used for plotting the type curve (Figures 1, 2, and 3), the observed curve can be fitted to the type curve in only one position. Then, from this fit, the value of W(u) and the corresponding value of u may be determined from the type curve for any selected point on the graph, which can be used in conjunction with the observed values for that point to determine the transmissibility T and the storage coefficient S (Wenzel, 1942, pp. 88-89). The selected point is usually called the "match point."

The transmissibility is then computed by the formula

$$T = \frac{114.6\ Q}{s} W(u) \qquad (6)$$

and the storage coefficient is computed by

$$S = \frac{u\ T\ t}{1.87r^2} \qquad (7)$$

Equations (6) and (7) are obtained from Equations (1a) and (2) solving for T and S.

Cooper and Jacob Method. A second form of solution was developed by Cooper and Jacob (1946, pp. 526-534). They noted that when u is small, the expression

$$W(u) = -0.577216 - \ln. u + u - \frac{u^2}{2.2!} + \ldots. \quad (3a)$$

is equivalent without any appreciable error to

$$W(u) = -0.5772 - \ln. u \quad (3b)$$

As they expressed Equations (1) and (1a) in the basic English units, that is, in feet, cubic feet per second, cubic feet per second per foot, and seconds, Equation (1a) can be rewritten:

$$s = \frac{Q}{4\pi T}(-0.5772 - \ln. u) \quad (1b)$$

and

$$s = \frac{Q}{4\pi T}\left(\ln. \frac{1}{u} - \ln. e^{0.5772}\right) \quad (1c)$$

substituting

$$u = \frac{r^2 S}{4Tt} \quad (2a)$$

we have

$$s = \frac{Q}{4\pi T} \ln. \frac{4Tt}{r^2 S e^{0.5772}}$$

or

$$s = \frac{Q}{4\pi T} \ln. \frac{2.25\ Tt}{r^2\ S} \quad (8)$$

Equation (8) can also be rewritten:

$$s = \frac{2.303\ Q}{4\pi T}\left(\log. t - \log. \frac{r^2 S}{2.25T}\right) \quad (9)$$

The only variables in this equation are the drawdown s and the time t. As we are at a certain observation well, r is constant.

Careful observation of Equation (9) will show that it represents the equation of a straight line like

$$y = a(x-c)$$

in which the slope of the straight-line plot is represented by the quantity on the outside of the bracket and the intercept of the straight line on the zero-drawdown line is represented by the second term without the bracket.

As T is the only unknown in the quantity representing the slope, the coefficient of transmissibility is readily determined from a semi-logarithmic plot of observed data by equating the slope of the plot with the corresponding quantity in Equation (9) and solving for T.

After T is determined, the only unknown remaining in the term representing the intercept will be S. Then, the storage coefficient can be computed by equating the intercept of the plot with the corresponding term, and solving for S.

Equating the quantity outside the brackets in Equation (9) with the slope of the graph we have

$$\frac{2.303\,Q}{4\pi T} = \frac{s}{\log.\, t} \qquad (10)$$

If Equation (10) is solved for T and for convenience log. t is made the unity by having it represent one logarithmic cycle, then

$$T = \frac{2.303\,Q}{4\pi\Delta s} \qquad (11)$$

Equating the second term in the bracket in Equation (9) with the intercept of the straight line on the zero-drawdown line, and solving for the storage coefficient, we have

$$S = \frac{2.25\,T\,t_0}{r^2} \qquad (12)$$

where t_0 is the value of t at the intercept.

In equations (11) and (12), T is the transmissibility expressed in cubic feet per second per foot, Q is the well discharge in cubic feet per second, Δs is the change in drawdown in feet for one log cycle of t in seconds, r is the distance in feet from the pumping well to the satellite well, and t_0 is the value of t in seconds at the intercept of the straight line with the zero-drawdown line.

With the units most commonly used in field practice the above equations are expressed:

$$T = \frac{264\ Q}{\Delta s} \qquad (11a)$$

and

$$S = \frac{2.1\ T\ t_0}{r^2} \times 10^{-4} \qquad (12a)$$

in which

T is the transmissibility in gallons per day per foot (gpd/ft),

Q is the discharge or pumping flow rate from the well in gallons per minute (gpm),

Δs is the change in drawdown in feet of the straight line for one logarithmic cycle of t in minutes,

t_0 is the value of t in minutes at the intercept of the straight line with the zero drawdown line and

r is the distance in feet from the pumping well to the satellite well.

In this method it may be seen from Equation (3a) that if u is less than 0.01

$$W(u) = -0.5772 - \ln u \qquad (3b)$$

with an error of less than 1 per cent.

The time-drawdown plotting or time-drawdown graph, as it is also usually called, will be straight lines only where $\frac{1}{t}$ is small so that u is small. A measurement of drawdown that is made too soon after the discharge is begun will not plot on the straight line, but on a curve asymptotic to it.

There are some other factors, besides, that divert the data collected from plotting in a straight line. Among these factors are partially delayed discharge of water in storage with the decline in head, reaching areas of recharge, and boundary effects. These last factors affect not only the straightness of the plotted line in this method but also the smoothness of the Theis curve.

Brown (1953, pp. 864-866) has clearly indicated the impermeable-barrier or boundary effect and the recharge effect on the drawdown test data plot.

Walton (1960, p. 22) has thoroughly discussed the application and limitation of these two methods, especially of the straight-line method, and he states: "However, the straight-line method is not applicable in some cases and it supplements, rather than supersedes, the more complicated type-curve method" (Theis method).

Cooper and Jacob Method with Modification by Lohman. The most serious error in the determination of aquifer constants with the "Straight-Line," "Semilog-Plot" or Cooper and Jacob method is in the determination of the storage coefficient. This error is caused by the inaccuracy in finding the intercept of the straight line with the line of zero drawdown. This inaccuracy is especially great in those cases of high transmissibility and therefore of a flat slope in the straight line. In these cases a slight change in the slope of the line best adapted to the plot may lead to errors of one or more cycles in the time scale. Each cycle of deviation in the time scale will represent a value for the storage coefficient ten times greater or smaller in accordance with the sign of the deviation.

A method has been developed by Lohman (1957, 6 pp.) for determining the coefficient of storage from straight-line plots without extrapolation. In the case of straight-line plots of drawdowns "s" versus $\log_{10}t$ or $\log_{10}t/r^2$, the storage coefficient S can be calculated by the expression:

$$S = \frac{2.25\ T\ {}^{t}/{r^2}}{\log^{-1}\left[-\frac{s}{\Delta s}\right]} \qquad (13)$$

which in units commonly used in field practice becomes:

$$S = \frac{2.1\ T\ {}^{t}/{r^2}}{\log^{-1}\left[-\frac{s}{\Delta s}\right]} \times 10^{-4} \qquad (13a)$$

where

S is the storage coefficient expressed as a fraction of unity,

T is the transmissibility in gallons per day per foot,

r is the distance in feet from the pumping well to the point of water-level observation,

t is the time in minutes of any point conveniently chosen on the straight line plot,

s is the corresponding drawdown in feet of the point chosen for t.

It should be pointed out that the negative sign in the bracketed term reflects the fact that drawdown increases with time. Thus in equations (13) or (13a) when substituting for Δs, which is the change in drawdown over one logarithmic cycle of t, the numerical value should be prefixed with a minus sign to identify properly the negative slope of the straight line plot. The two negative signs then combine to make the bracketed term positive.

The development of equation (13) was made by Lohman (1957, p. 3) based on work by Jacob and Lohman (1952, p. 567) using the straight-line method and its basic equation (8).

VI. JAEGER METHOD

J. C. Jaeger (1959, pp. 561-564) has proposed this new method for the determination of aquifer constants: transmissibility and storage coefficient, based again in the analogy between heat conduction and ground water flow. What he has, in effect, proposed is a practical numerical solution for the Theis non-equilibrium formula.

In his paper Jaeger states:

> "One of the most satisfactory methods of measuring the thermal conductivity of powders consists of heating a long straight wire and measuring the temperature variation at a point some distance from it, similarly, the properties of an aquifer may be measured by pumping from a well and measuring the drawdown at a second well. In both cases the theory is that of a continuous line source in radial flow (Carslaw and Jaeger, 1959, p. 261) and a simple theoretical solution is available. In the hydrological case, reduction of the results is done (Brown, 1953, pp. 844-866) either by a process of curve fitting (Theis method) or by the use of the logarithmic asymptote of the theoretical solution (Cooper and Jacob method); in conduction of heat, the latter method has always been employed; in both cases it implies that relatively long times must be used and it does not provide a very accurate value for the coefficient of storage or, in the thermal case, the diffusivity.
>
> " . . . The method of reduction described below is available for all times, and the writer has found it particularly useful for measurements of thermal conductivity; it is set out here in case it may also be useful in that context."

The non-equilibrium formula can be written:

$$s = \frac{114.6\ Q}{T} W(u) \qquad (1a)$$

where

$$W(u) = -0.5772 - \ln.u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} \ldots \qquad (2a)$$

and

$$u = \frac{1.87r^2S}{T\ t} \qquad (2)$$

Then we have:

$$W(u) = W\left(\frac{1.87r^2S}{T\ t}\right) = \qquad (3a)$$

$$= -0.5772 - \ln.\ \frac{1.87r^2S}{T\ t} + \frac{1.87r^2S}{T\ t} - \frac{(1.87r^2S)^2}{2 \cdot 2!\ (Tt)^2} \qquad (3b)$$

As stated in Chapter IV, values of W(u) for values of u between 10^{-15} and 9.9 are given by published tables (Wenzel, 1942, p. 89) and are also graphically given in Figures 1, 2, and 3.

The method suggested is as follows: "let t_o be any convenient time and let N be any number (in practice N is conveniently taken to be 2); then if $s(t_o)$ and $s(Nt_o)$ are the values of the drawdown at times t_o and Nt_o, it follows from Equations (1a) and (2) that"

$$\frac{s(Nt_o)}{s(t_o)} = \frac{W\left(\frac{1.87r^2S}{TNt_o}\right)}{W\left(\frac{1.87r^2S}{T\ t_o}\right)} \qquad (14)$$

"Here $s(Nt_o)$ / $s(t_o)$ is known experimentally and so $(1.87r^2S/TNt_o)$ can be read from a table or graph of W(u) / W(Nu)."

This last paragraph can be more easily understood if Equation (14) is rewritten:

$$\frac{s(Nt_o)}{s(t_o)} = \frac{W\left(\frac{1.87r^2S}{TNt_o}\right)}{W\left(\frac{1.87r^2S}{T\ t_o}\right)} = \frac{W\left(\frac{u_o}{N}\right)}{W(u_o)} = \frac{W(u)}{W(Nu)} \qquad (15)$$

remembering of course that W(u) is the well function of u, which corresponds to the drawdown $s(Nt_o)$.

Then, with the value obtained experimentally for $s(Nt_o)/s(t_o)$ we enter a table or graph of $W(u)/W(Nu)$ vs. u using the same value for $W(u)W(Nu)$ as $s(Nt_o)/s(t_o)$, as from Equation (15)

$$\frac{s(Nt_o)}{s(t_o)} = \frac{W(u)}{W(Nu)}$$

Entering such a table or graph with that value, we obtain the u value that corresponds to $s(Nt_o)$. With the value of u so obtained we can get the corresponding value of $W(u)$ from a $W(u)$ vs. u table (Wenzel, 1952, p. 89) or from a $W(u)$ vs. u graph such as those shown in Figures 1, 2, and 3.

With the value of $W(u)$ the transmissibility T can be computed from Equation 6

$$T = \frac{114.6\ Q}{s} W(u) \qquad (6)$$

using for s the value of $s(Nt_o)$.

Once T has been computed, the storage coefficient S is obtained from Equation (7)

$$S = \frac{u\ T\ t}{1.87r^2} \qquad (7)$$

using of course for t, the value of Nt_o.

$W(u)/W(2u)$ vs. u graphs have been drawn by this writer for values of u between 1×10^{-15} and 1.2 that correspond to $W(u)/W(2u)$ values between 1.0208 to 6.0. These graphs are shown on Figures 4, 5, and 6.

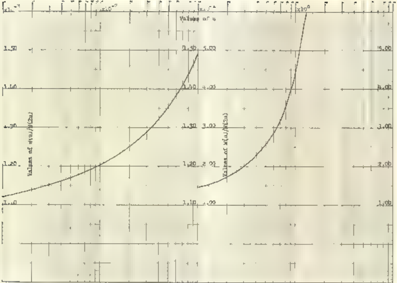


Fig 4. Values of $W(u)/W(2u)$ for Values of u between 1×10^{-3} to 1.3

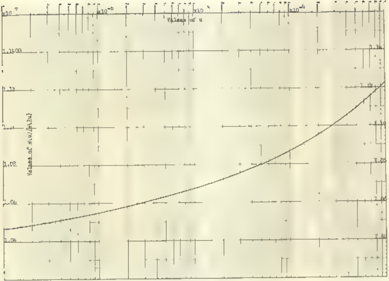
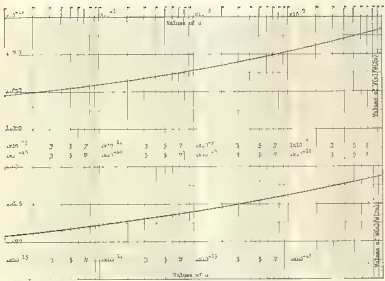


Fig   Values of W(u)/ln(2u) for values of u between 1x10⁻⁷ to 1x10⁻³



[illegible] values of [illegible] for [illegible] values of u between [illegible]

It can be very easily seen that with the Jaeger method it is possible to obtain one value for the transmissibility T and one value for the storage coefficient S for each pair of drawdowns $s(Nt_o)$ and $s(t_o)$ corresponding to t_o and Nt_o, t_o being any time after pumping began. In actual practice N is usually taken as 2 and therefore, it is possible to obtain one value for T and S for each pair of drawdowns corresponding to any pair of time values which hold the relation of 1:2.

Theoretically, the method suggests that with very short periods of pumping, even with only some few minutes, both aquifer constants can be easily determined. This would hold true if the aquifer would fulfill all the assumptions on which the non-equilibrium formula is based.

In practice, however, aquifers deviate from the mentioned assumptions. In many cases this deviation is small, and some time after pumping has started the method can be applied with reliable results. In other cases, a longer period of time is needed, and still, in some other cases this initial period has not been yet completed when other deviations from the general assumptions or some other factors affect the results obtained.

As may be easily understood, those deviations are not a deficiency of the proposed method or of the non-equilibrium formula but rather are the effect of the complexity of the aquifer. The same difficulties are present, of course, with the other two methods. However, their presence is not so easily identifiable as with the Jaeger method, and not to have a clear knowledge of the presence of those factors is quite dangerous, as it may lead to the use of a value without consideration of its worth or limitations.

The procedure developed by Garcia Bengochea (1960, p. 39) for the application of the Jaeger method gives a clear picture of the reliability and deviations of the values so obtained for both aquifer constants. This holds true even for the unexperienced observer and under rather complex aquifer conditions.

When aquifer conditions closely resemble the assumptions of the non-equilibrium formula, the proposed method is fast and accurate. Under these conditions the writer thinks that it is also somewhat more reliable than the other two methods because the observer does not depend on graphical solutions. Although some graphs are also used with this procedure for the application of the Jaeger method, they are used as an aid for the computing work that is purely numerical, and also, as an aid to express the results obtained.

The procedure developed by Garcia Bengochea (1960, p. 39) is as follows:

a. With the drawdown data collected from the observation well a "Natural Time-Drawdown Graph" is drawn by plotting on natural paper the observed drawdowns against the time since pumping was started. This graph is an aid for the application of the method and can be substituted by a tabulation of the observed drawdowns from a water level recorder chart or from direct observations taken at the time intervals indicated in paragraph b.

b. A tabulation is then prepared as that shown in Table II in which the method is applied for different small time intervals. In Table II twenty-minute time intervals have been used. The drawdowns

TABLE II

ACUEDUCTO DE MARIANO, CUBA
PUMPING TEST OF WELL 13
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
DRAWDOWN DATA FROM SATELLITE WELL N-3

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
10	1	0.030					
20	2	0.047	1.57	1.3×10^{-1}	1.59	19,400,000	0.019
20	1	0.047					
40	2	0.071	1.51	1.1×10^{-1}	1.74	14,000,000	0.024
40	1	0.071					
80	2	0.105	1.479	9.5×10^{-2}	1.86	10,100,000	0.029
60	1	0.089					
120	2	0.131	1.472	9.2×10^{-2}	1.90	8,280,000	0.035
80	1	0.105					
160	2	0.154	1.467	9.0×10^{-2}	1.92	7,120,000	0.039
100	1	0.119					
200	2	0.177	1.488	9.9×10^{-2}	1.83	5,900,000	0.045
120	1	0.131					
240	2	0.198	1.511	1.1×10^{-1}	1.74	5,010,000	0.051
140	1	0.144					
280	2	0.220	1.528	1.2×10^{-1}	1.66	4,310,000	0.055
160	1	0.154					
320	2	0.241	1.565	1.3×10^{-1}	1.59	3,760,000	0.060
180	1	0.166					
360	2	0.262	1.518	1.2×10^{-1}	1.66	3,620,000	0.060
200	1	0.177					
400	2	0.282	1.592	1.4×10^{-1}	1.52	3,080,000	0.067
220	1	0.188					
440	2	0.303	1.611	1.5×10^{-1}	1.46	2,750,000	0.070
240	1	0.198					
480	2	0.324	1.636	1.6×10^{-1}	1.41	2,490,000	0.073
260	1	0.209					
520	2	0.343	1.640	1.6×10^{-1}	1.41	2,350,000	0.075
280	1	0.220					
560	2	0.362	1.644	1.6×10^{-1}	1.41	2,220,000	0.076
300	1	0.230					
600	2	0.379	1.648	1.7×10^{-1}	1.36	2,050,000	0.080
320	1	0.241					
640	2	0.396	1.643	1.6×10^{-1}	1.41	2,030,000	0.080
340	1	0.251					
680	2	0.411	1.635	1.6×10^{-1}	1.41	1,960,000	0.082

TABLE II - Continued

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
360	1	0.262					
720	2	0.425	1.622	1.6×10^{-1}	1.41	1,890,000	0.083
380	1	0.272					
760	2	0.439	1.614	1.6×10^{-1}	1.41	1,830,000	0.085
400	1	0.282					
800	2	0.451	1.599	1.5×10^{-1}	1.46	1,850,000	0.085
420	1	0.292					
840	2	0.462	1.582	1.4×10^{-1}	1.52	1,880,000	0.085
440	1	0.303					
880	2	0.473	1.561	1.3×10^{-1}	1.59	1,920,000	0.084
460	1	0.313					
920	2	0.484	1.548	1.3×10^{-1}	1.59	1,870,000	0.086
480	1	0.324					
960	2	0.495	1.528	1.2×10^{-1}	1.66	1,910,000	0.084
500	1	0.333					
1000	2	0.505	1.517	1.2×10^{-1}	1.66	1,850,000	0.085
520	1	0.343					
1040	2	0.516	1.502	1.1×10^{-1}	1.74	1,930,000	0.083
540	1	0.353					
1080	2	0.527	1.493	1.0×10^{-1}	1.82	1,970,000	0.082
560	1	0.362					
1120	2	0.537	1.483	9.7×10^{-2}	1.85	1,970,000	0.082
580	1	0.371					
1160	2	0.547	1.474	9.3×10^{-2}	1.89	1,970,000	0.081
600	1	0.379					
1200	2	0.557	1.471	9.2×10^{-2}	1.90	1,950,000	0.082
620	1	0.388					
1240	2	0.566	1.458	8.7×10^{-2}	1.95	1,970,000	0.081
640	1	0.396					
1280	2	0.576	1.453	8.5×10^{-2}	1.97	1,950,000	0.081
660	1	0.403					
1320	2	0.585	1.452	8.5×10^{-2}	1.97	1,920,000	0.083
680	1	0.411					
1360	2	0.594	1.446	8.2×10^{-2}	2.00	1,920,000	0.082
700	1	0.418					
1400	2	0.603	1.442	8.1×10^{-2}	2.02	1,910,000	0.083
720	1	0.425					
1440	2	0.611	1.437	7.9×10^{-2}	2.04	1,910,000	0.083

TABLE II - Continued

$Q = 4980$ gpm $\quad r = 984$ feet

$$T = \frac{114.6\ Q\ W(u)}{s} = \frac{114.6 \times 4980 \times W(u)}{s} = 571{,}000\ \frac{W(u)}{s}$$

$$S = \frac{u\ T\ t}{1.87\ r^2} = \frac{u\ T\ t'}{1.87 \times 0.968 \times 10^6 \times 1440} = \frac{u\ T\ t'}{2.61} \times 10^{-9}$$

for the third column are obtained from the natural time-drawdown graph or from a drawdown tabulation as indicated in paragraph a. The values of u are obtained by entering with the s/s_o value a table or graph that gives $W(u)/W(2u)$ vs. u. Figures 4, 5, and 6 show such a graph. Then the values of $W(u)$ are found from a $W(u)$ vs. u table (Wenzel, 1942, p. 89) or from a $W(u)$ vs. u graph as that shown in Figures 1, 2, and 3. Once the values of u and $W(u)$ are known, T and S are easily computed with Equations (6) and (7) of Chapter 6.

c. If the observer wishes to have a better view of the various results so obtained, another graph is drawn by plotting on natural paper the values found for T and S against the time since pumping was started. Figure 7 shows the two curves obtained using the values of T and S given in Table II.

The differences among the values found for T and S during the first 600 minutes (10 hours) of pumping clearly show that some assumption or assumptions made for the non-equilibrium formula did not hold true for this part of the test. This deviation is generally believed to be produced because water taken from storage by the decline in water level is not drained instantaneously. This deviation can also be identified in a Cooper and Jacob time-drawdown graph by the downward diversion of the first points plotted, but, in the opinion of this writer, it is not as clearly identifiable as with the Jaeger method. As Walton explains (Walton, 1960, Part I, p. 50) "The non-equilibrium formula does not accurately define drawdown until adjustment of flow occurs between permeable zones and until the effects of delayed

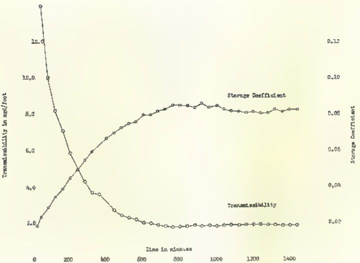


Fig. 7.-Values Obtained for and S, Acueducto de Marianao, Well 13, Drawdowns in Satellite Well R-3

discharge of water in storage become small. The initial transitional period during which drawdown data deviate from theory may be less than a minute, several hours, or even days in duration depending upon the difference between ideal and actual aquifer conditions."

d. According to Figure 7 the deviation effect seems to be overcome after 600 minutes (10 hours) of pumping; then the computations made from the 300-600 minute observation until the end of the test can be averaged as shown on Table III. The average values for T and S so obtained should represent the transmissibility and the storage coefficient of the formation where the well is located.

TABLE III

ACUEDUCTO DE MARIANAO, CUBA
PUMPING TEST OF WELL 13
JAEGER METHOD SELECTED COMPUTATIONS FOR T AND S
DRAWDOWN DATA FROM SATELLITE WELL N-3

Time min.		T gpd/foot	S
300 - 600 to 720 - 1440		See Table II for corresponding values of T and S	
		$\sum$ 42,410,000	$\sum$ 1.822
	Average	1,928,000	0.0828
	Rounded	1,930,000	0.083

The data and results of the previous example, Test Well 13 of the Acueducto de Marianao, correspond to a formation which seems to agree with the assumptions of the non-equilibrium formula at the location of the observation well some time after pumping began. The agreement, however, is not so definite in some other cases.

Figure 8 shows the values obtained for T and S for the formation around Well 15, City of Titusville, Florida. Drawdown data for this test was collected from a satellite well located ten feet away from the pumping well, which was pumped at the rate of 210 gallons per minute. Values for T and S agree within reasonable limits for those observations made in the time period between eight and thirty minutes after pumping began. After 40 minutes of pumping, the values of the transmissibility kept increasing and the values for the storage coefficient started decreasing. This phenomenon has been found to typify leaky aquifers where the main aquifer being pumped is recharged from underlying or overlying formations.

The geology of the area, driller's logs, water level information and common sense shall permit the observer to evaluate these circumstances. It must be remembered that, by definition, the storage coefficient, when expressed dimensionless, cannot be greater than unity. It should also be remembered that, although rigid limits cannot be established, storage coefficients range for water-table conditions from 0.05 to 0.30 and for artesian aquifers may range from 0.00001 to 0.001 (1×10^{-5} to 1×10^{-3}) (Ferris, 1962, pp. 76-78).

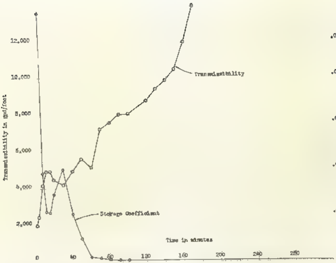


Fig. 8.-Values Obtained for T and S, City of Titusville, Well 15, Drawdowns in Satellite Well

VII. LEAKY AQUIFER CONDITIONS

Vertical leakage or recharge is probably the most common deviation from the assumptions of the non-equilibrium formula showed by aquifers in Florida. This case is that of an elastic artesian aquifer which is recharged by vertical leakage through overlying or underlying confining strata. In most instances of artesian aquifers, the confining bed or beds are semipermeable rather than impermeable and only retard the movement of ground water instead of preventing it. This retardation of ground-water movement is sufficient in many cases to validate the assumption of impermeable confining beds in the non-equilibrium formula. However, there are also many cases when vertical leakage through the semi-confining bed will prevent the indiscriminate use of the non-equilibrium formula, as this recharge will tend to establish an equilibrium.

According to Ferris (1962, p. 111) the first detailed analysis and solution of leaky-aquifer problems was done by Glee (1930) and later supplemented by Steggewentz and Van Nes (1939, pp. 561-563). Work by Jacob (1946, pp. 198-205), and by Hantush and Jacob (1954, pp. 917-936; 1955, pp. 95-100) resulted in the solution of the non-steady distribution of drawdown caused by pumping a well at a constant rate from an aquifer. The aquifer is assumed to be: (1) infinitely effective, (2) perfectly elastic, (3) uniform in thickness, and

(4) replenished by leakage which takes place in proportion to the drawdown. This solution is expressed by the expression.

$$s = \frac{Q}{4\pi T} W(u,x) \qquad (16)$$

where Q, T and u have the same significance as in the non-equilibrium formula and,

$$x = \frac{r}{B} = \frac{r}{\sqrt{\frac{T\,m'}{P'}}} = r\sqrt{\frac{P'}{T\,m'}} \qquad (17)$$

in which r has the same significance as in the non-equilibrium formula, and

P' is the coefficient of vertical permeability of the semipermeable confining bed,

m' is the thickness of the semipermeable confining bed,

$\frac{P'}{m'}$ is also designated as the "leakage coefficient" or "leakance" (Hantush, 1956, p. 702).

Hantush (1956, p. 702) calls W(u,x) the "well function for leaky systems:" it can be expressed:

$$W(u,x) = \int_{u}^{\infty} \left(\frac{1}{y}\right) \exp.\left(-y - \frac{x^2}{4B^2 y}\right) dy \qquad (18)$$

$$= 2K_o(x) - \int_{\frac{x^2}{4u}}^{\infty} \left(\frac{1}{y}\right) \exp.\left(-y - \frac{x^2}{4y}\right) dy \qquad (18a)$$

The symbol $K_o(x)$ is the modified Bessel function of the second kind of the zero order. Values of $K_o(x)$ for values of x between 10^{-2} and 9.9 are given by Ferris (1962, p. 115). For values of x ranging between 0 and 10^{-2}, $K_o(x)$ can be determined by the expression Ferris (1962, p. 116):

$$K_o(x) = 0.116 - 2.303 \log_{10}(x) \qquad (19)$$

Equation (16) defines the solution for the transient state or state of non-steady distribution of drawdown. The steady-state distribution of drawdown, maximum drawdown, in the vicinity of the well is obtained from Equation (16) by letting t approach infinity, and then (Ferris, 1962, p. 112)

$$s_m = \frac{Q}{2\pi T} K_o(x) \qquad (20)$$

where s_m is the maximum drawdown at the distance r and the other symbols have the same significance as before.

The development of the solutions for leaky aquifer conditions assumes (Ferris, 1962, p. 111) that:

1. The artesian aquifer is bounded above or below by a semipermeable confining bed.

2. The aquifer, when pumped is supplied by leakage through the semipermeable confining bed, the leakage being proportional to the drawdown in the main aquifer.

3. The aquifer and confining bed are independently homogeneous and isotropic.

4. The water level in the secondary aquifer supplying water through the semipermeable bed is maintained at or very near static level through the interval of pumping.

Because of the mathematical difficulties encountered in applying either equation (16) or (20), several investigators have developed approximate graphical solutions to obtain the values of T, S, and P' or $\frac{P'}{m'}$ when pumping test data are available.

Cooper (1963) has developed a method to determine the transmissibility and the storage coefficient of the main aquifer and the leakage coefficient or "leakance" of the confining bed with non-steady drawdown data by an extension of the Theis graphical method. It consists in superposing a plot of the drawdowns on a family of type curves. The curves can be used also to predict drawdowns when aquifer and confining-bed constants are already known.

The development and use of those "Type Curves for Nonsteady Radial Flow in an Infinite Leaky Aquifer" are described in Cooper's words (1963) as follows.

> "Type curves.—The type curves [Fig. 9] are computed from the solution for nonsteady radial flow in an infinite leaky aquifer derived by Hantush and Jacob (1955):
>
> $$s/(Q/4\pi T) = 2K_o(2v) - \int_{v^2/u}^{\infty} (1/y)\exp(-y - v^2/y)dy \quad (2)$$
>
> where $u = r^2S/4Tt$
> t = time since the discharge began
> S = the coefficient of storage of the aquifer
>
> $\left[v = \frac{x}{2}\right]$
>
> and other terms are as previously defined. Hantush and Jacob gave two series expressions for the formal solution, one convenient for long times and one for short times. The solution for long times is:
>
> $$s/(Q/4\pi T) = 2K_o(2v) - I_o(2v)W(v^2/u) + \left[\exp(-v^2/u)\right]\Big[x(u) + 0.5772 + \log_e u - u + (u/v^2)\left[I_o(2v) - 1\right] - u^2\sum_{n=1}^{\infty}\sum_{m=1}^{n}\frac{(-1)^{n+m}\,(n-m+1)!}{(n+2)!^2}(v^2/u)^m u^n\Big] \quad (3)$$

and that for short time is:

$$s/(Q/4\pi T) = I_o(2v)W(u) - e^{-u}\Big[0.5772 + \log_e(v^2/u) + W(v^2/u) - v^2/u + \big[I_o(2v) - 1\big]\ /u\Big] + (e^{-u}/u^2)\sum_{n=1}^{\infty}\ \sum_{m=1}^{n}\ \frac{(-1)^{n+m}\ (n - m + 1)!\ (v\ /u)^n u^m}{(n + 2)!^2} \quad (4)$$

where I_o = the modified Bessel function of the first kind of order zero

$$W(x) = -Ei(-x) = \int_x^{\infty} (e^{-x'}\ /x')dx'$$

and other terms are as previously defined.

"The solution may be written:

$$s = (Q/4\pi T)\ L(u,v)$$

where L(u,v) represents the right-hand side of equations 2, 3, or 4. In the limiting case wherein P' and hence v approach zero, L(u,v) approaches W(u) and the solution becomes:

$$s = (Q/4\pi T)\ W(u) \qquad (5)$$

which is the familiar Theis nonequilibrium formula (Wenzel, 1942, p. 88).

"The type curves, which are plots of L(u,v) versus 1/u, consist of two families, one of which v is used as the parameter and another in which v^2/u is used as the parameter. The parent curve of both families, labeled W(u), is the Theis nonequilibrium type curve. An undashed curve, for which v is the parameter, corresponds to a plot of drawdown versus time t at some constant distance r from the discharging wall, whereas a dashed curve, for which v^2/u is the is the parameter, corresponds to a plot of drawdown versus $1/r^2$ at some constant time t. Either set of curves may be used to compute the aquifer coefficients T, S, and P' /m'.

"By use of the undashed curves one may compute T, S, and P' /m' as follows: (1) plot a time-drawdown curve (s versus t/r^2, r constant) for each observation well on logarithm paper of the same scale as that of the type curves; (2) keeping the coordinate axes of the two sheets parallel, superpose the data plot on the undashed type curves to obtain a good match, observing that the early data from each well should approach the limiting curve labeled $W(u)$ and the later data should fall between two of the curves labeled $v = 2.2$, $v = 2.0$, etc., or along one of them; (3) as in the Theis graphical method, choose a convenient match point anywhere on the graph sheet and from its coordinates on the data and type curves respectively note the values of s, t/r^2, $L(u,v)$, and $1/u$; (4) for each observation well estimate the value of v corresponding to the distance r from the positions of the data plot between two of the limbs on the type curve. From the figures so obtained, the formation constants may be computed from the equations:

$$T = \frac{Q}{4\pi} \frac{L(u,v)}{s} \tag{6}$$

$$S = 4T \frac{(t/r^2)}{(1/u)} \tag{7}$$

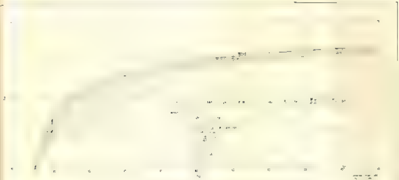
$$\frac{P'}{m'} = 4T \frac{v^2}{r^2} \tag{8}$$

"Equations 6, 7, and 8 are for consistent units of length and time. It is customary in the Geological Survey to express r in feet, t in days, Q in gallons per minute, T in gallons per day per foot, P' in gallons per day per square foot, and m' in feet, S being dimensionless. For these inconsistent units equations 6 and 7 become:

$$T = 114.6Q \frac{L(u,v)}{s} \tag{9}$$

$$S = \frac{T(t/r^2)}{1.87(1/u)} \tag{10}$$

and equation 8 remains unchanged."



TYPE CURVES FOR NON-STEADY RADIAL FLOW IN AN INFINITE LEAKY AQUIFER

Fig. 9.

VIII. EXPERIMENTAL WORK

The three methods for the determination of aquifer constants with drawdown data from one or more satellite wells, described in detail in the previous chapters, have been applied in this work to drawdown data collected in the pumping well of tests performed by the writer. The methods are: Jaeger's, the Semilog-Plot of Cooper and Jacob and Theis' with the extension of Cooper's Type Curves whenever required. Results obtained have been evaluated by comparing them among themselves and also with results obtained by the application of the same methods to drawdown data from satellite well or wells.

Data from pumping tests in four different wells have been used in the present work. The first two tests were performed before this research was considered, however, enough data were available to permit a preliminary application of the above mentioned methods to drawdown data from the pumping well. The results from the application of these methods to data from the pumping well was then compared with those obtained from water-level observations in a satellite well or wells. The first two tests were at Acueducto de Marianao Well 13 and Compania Cervecera de las Antillas Well 2, both in the province of La Habana, Cuba.

Experience from the first two evaluations was then employed to program data collection for a third pumping test. This new test was at

Well 10A, Fort Pierce, Florida, where one satellite well was available. Results from the different methods and from both satellite and pumping well were compared.

Finally a fourth well was tested in Leesburg, Florida, Well 12, with no satellite well available. Data collection in the pumping well was programmed to produce the best possible information. This well was tested twice: one time before it was completely developed and the second time after the development work was completed. The three selected methods were used with drawdown data from both tests and the results from all methods and both tests evaluated.

The flow rate was determined in each case with pipe orifices with free discharge (Greve, 1949, 25 pp.). The hydraulic head on the orifice was recorded with a pressure-recorder calibrated in inches of water.

Water level observations in the satellite wells were made with Stevens and Leupold water level recorders, type F. Water level observations in the first two pumping wells were made with water-level indicators of the electrode type, M-Scope WL-250, manufactured by Fisher Research Laboratories. Water level observations in the pumping wells at Fort Pierce and Leesburg were made with a bubbler-system equipment similar to that developed by Nelson (1954, 30 pp.).

The bubbler-system equipment employed for water-level measurements in the last two pumping wells is schematically shown on Figure 10. Nitrogen gas was used instead of the carbon dioxide gas proposed by Nelson because nitrogen, being practically an inert gas will not affect the chemical composition of the water in the well. The U-tube on the

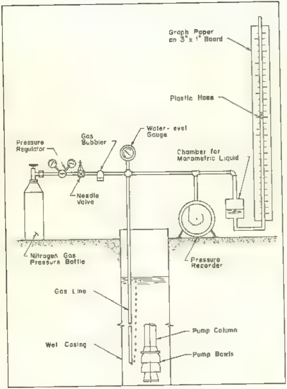


Fig. 10.-Schematic Diagram of Bubbler System

right hand side of Figure 10 has one leg formed by a manometric-liquid chamber, four inches in diameter and four inches high; the other leg is a transparent plastic hose, 3/16 inch in diameter and eight feet long. The plastic hose was attached to a wooden board, three inches wide and 66 inches long. Before the hose was attached to the board, the front face of the board was covered in each test with a strip of graph paper, ten divisions to the inch, three inches wide and 66 inches long.

The ratio between the cross-sectional area of the manometric-liquid chamber to that of the plastic hose was approximately 450.1. This fact permitted us to determine the differential head in the U-tube by reading only the plastic hose. The estimated error was of 0.0022 inch per inch of head differential and, of course, too small to be corrected.

The bubbler system, as described, permitted the writer to take water level observations as often as every second. This was done by simply plotting a mark on the graph paper at the same level the manometric level was at any time. Time was kept either by hearing a signal every second from a battery operated metronome or by reading a stop-watch every five seconds.

Because of the magnitude of the drawdowns in both wells, at Fort Pierce and Leesburg, mercury was employed as the manometric liquid. For tests where less drawdown is expected another manometric liquid of lower specific gravity may be used, and consequently the accuracy of the measurements is increased.

Figure 11 shows the bubbler-system instrumentation as assembled

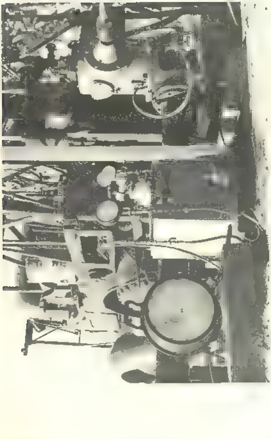


Fig. 11.--Bubbler-System Instrumentation Assembled in the Field

in the field. The recorder shown had two pens with two pressure-gaging elements. One pen recorded the piezometric head, in inches of water, on the flow-rate-measuring orifice. The other pen connected to the bubbler-system recorded the drawdown in the pumping well in feet of water.

It must be mentioned that the bubbler system was adopted in order to make fast and accurate water-level observations in the pumping well. This was found to be necessary after the evaluation made with the available data from the first two tests Well 13, Acueducto de Marianao and Well 2, Compania Cervecera de las Antillas. The evaluation of these two tests is included in the following Chapter IX: Preliminary Evaluations.

IX. PRELIMINARY EVALUATIONS

The first two evaluations of the determination of aquifer constants with water level observations in the pumping well were made with data available from two pumping tests performed by the writer some time before this research was considered. This was done in order to have a preliminary evaluation of the possibilities of such an idea.

Two pumping tests were selected in which regular and accurate water-level observations in the pumping well were available together with reliable results of the aquifer constants obtained from water-level observations in satellite wells. These two tests were Pumping Test of Well 13, Acueducto de Marianao, Cuba (Garcia Bengochea, 1957, 49 pp.) and Pumping Test of Well 4, Compania Cervecera de las Antillas, Cuba (Garcia Bengochea, 1958, 40 pp.).

A detailed description of the evaluation of each pumping test is given immediately following.

A. Pumping Test of Well 13, Acueducto de Marianao, La Habana, Cuba

Well 13 is one of the six new wells developed during 1957 and 1958 for the city of Marianao, in the province of La Habana (Havana), Cuba. The city of Marianao is just west of the city of Havana, and it covers to a great extent the most modern residential developments of what may be called the Greater Havana Area.

The new well field has a total capacity of 30 million gallons per day yielded from only six wells. This field has been developed in the Ariguanabo basin where a very rich aquifer was quite recently discovered.

At the site of well 13 the aquifer extends from 25 feet to 180 feet in depth, its thickness being, then, 155 feet. It is formed by cavernous limestone geologically classified as upper Oligocene.

All six wells penetrate the entire thickness of the aquifer and were drilled 30 inches in diameter. They are cased to the top of the limestone and open hole thereafter. A numbering system from 11 to 17 was used for these wells in order to differentiate them from the wells in another well field.

Well 13 was pumped for 24 consecutive hours at a rate of 4980 gpm (gallons per minute) and water levels were recorded in three observation wells eight inches in diameter and located respectively at 656, 820 and 984 feet. Figure 12 shows the deep-well pump and instrumentation in well 13 during its pumping test.

Values of the aquifer constants in the vicinities of well 13 and its satellites wells were determined by Garcia Bengochea (1957, p. 10) and are reproduced in Table IV. Determinations were made at that time by the Theis method and the Cooper and Jacob method. Results from the two methods and the three different satellite wells agree remarkably well for this type of determination and give an average value for the transmissibility T of 2.09 mgd foot (millions of gallons per day per foot) and for the storage coefficient S of 0.076.

Fig. 12 [illegible] oct of Wo. acueducto de Marlonan

TABLE IV

ACUEDUCTO DE MARIANAO, CUBA
PUMPING TEST OF WELL 13
VALUES OBTAINED FOR THE TRANSMISSIBILITY AND THE STORAGE COEFFICIENT
USING WATER-LEVEL OBSERVATIONS IN SATELLITE WELLS

(Garcia Bengochea, 1957, p. 10)

Method	Distance to Pumping Well feet	Transmissibility T mgd/foot	Storage Coef. S -
Theis	656	2.33	0.039
Cooper and Jacob	656	2.07	0.049
Theis	820	2.01	0.110
Cooper and Jacob	820	2.03	0.105
Theis	986	2.00	0.081
Cooper and Jacob	986	2.08	0.071

Aquifer constants were determined later by the Jaeger method with the same drawdown data of the satellite well at 986 feet (Garcia Bengochea, 1960, p. 45). The value determined for T was 1.93 mgd/foot and that for S was 0.08.

Figure 13 shows the natural time-drawdown graph of pumping well 13; and Table V shows the Jaeger-method computations for time intervals of one minute after pumping started with drawdowns in the pumping well.

An analysis of Table V indicates that there are only two values of the storage coefficient which are really meaningful: the values corresponding to the 2.4 and 3.6 minute intervals which are 0.009 and 0.016. The corresponding values for the Transmissibility for those two sets of observations are respectively 2.43 and 2.84 mgd/foot: their average 2.63 mgd/foot seems to agree with the values determined for T with the data from the satellite wells. Prior to two minutes of pumping the value obtained for S was greater than unity, a value which cannot be possible. After four minutes values obtained for the storage coefficient were so small as to be meaningless in the kind of aquifer we were dealing with. After four minutes of pumping, the values determined for T and S, vary in a way which is typical of conditions of recharge.

Figure 14 shows the Semilog plot of the drawdowns in the pumping well 13 and also the computations for T and S in accordance with Cooper and Jacob method and its modification by Lohman (1957, 6 pp.). T is determined as 2.34 mgd/foot and S is 0.014. These two values agree very closely with those obtained by the Jaeger method in the pumping well. Very significantly in the graph is also the fact that drawdowns taken

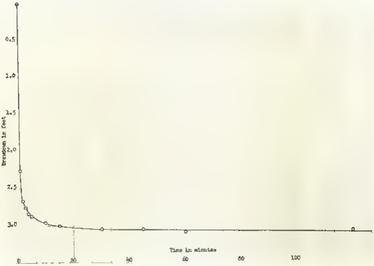


Fig. 13.-Natural Time-Drawdown Graph, Acueducto de Marianao, Well 13, Drawdowns in Pumping Well

TABLE V

ACUEDUCTO DE MARIANAO, CUBA
PUMPING TEST OF WELL 13
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
DRAWDOWN DATA IN THE PUMPING WELL

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
1	1	2.25					
2	2	2.66	1.182	6.4×10^{-3}	4.48	0.976×10^{6}	3.0×10^{0}
2	1	2.66					
4	2	2.83	1.062	3.9×10^{-6}	11.88	2.43×10^{6}	9.0×10^{-3}
3	1	2.75					
6	2	2.89	1.051	3.9×10^{-7}	14.18	2.84×10^{6}	1.6×10^{-3}
4	1	2.83					
8	2	2.93	1.036	1.3×10^{-9}	19.88	3.92×10^{6}	9.7×10^{-6}
5	1	2.87					
10	2	2.95	1.028	5.0×10^{-12}	25.44	4.99×10^{6}	5.9×10^{-8}
6	1	2.89					
12	2	2.97	1.028	5.0×10^{-12}	25.44	4.96×10^{6}	7.0×10^{-8}
7	1	2.91					
14	2	2.99	1.028	5.0×10^{-12}	25.44	4.92×10^{6}	8.2×10^{-8}
8	1	2.93					
16	2	3.01	1.028	5.0×10^{-12}	25.44	4.90×10^{6}	9.3×10^{-8}
9	1	2.94					
18	2	3.02	1.028	5.0×10^{-12}	25.44	4.88×10^{6}	1.0×10^{-7}
10	1	2.95					
20	2	3.02	1.023	2.5×10^{-14}	30.74	5.89×10^{6}	7.0×10^{-11}
11	1	2.96					
22	2	3.03	1.023	2.5×10^{-14}	30.74	5.87×10^{6}	7.6×10^{-11}
12	1	2.97					
24	2	3.03	1.021	1.3×10^{-15}	33.70	6.43×10^{6}	4.8×10^{-11}
13	1	2.98					
26	2	3.04	1.021	1.3×10^{-15}	33.70	6.42×10^{6}	5.1×10^{-11}

Q = 5050 gpm   d = 30 inches   r = 1.25 foot

$$T = 114.6\ Q\ \frac{W(u)}{s} = 114.6 \times 5050 \times \frac{W(u)}{s} = 0.579 \times 10^{6}\ \frac{W(u)}{s}$$

$$S = \frac{u\ T\ t}{1.87\ r^2} = \frac{u\ T\ t'}{1.87 \times (1.25)^2 \times 1440} = \frac{u\ T\ t'}{4.22} \times 10^{-3}$$

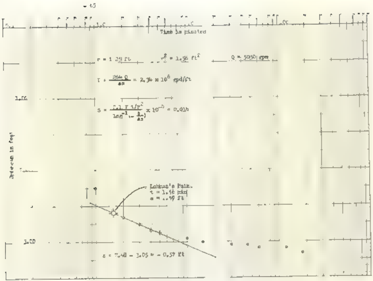


Fig 14. Semilog [illegible] Drawdown Graph [illegible] de [illegible], Well 13, Drawdown in Pumping Well

five minutes after pumping started showed a deviation from the straight line which is also typical of recharge effects.

It must be pointed out that well 13, together with the other five water supply wells for Acueducto de Marianao, was developed around a shallow lake called "Laguna de Ariguanabo." A bed of red and black clay ranging in depth between ten and thirty feet separated the lake from the main aquifer. Pumping from well 13 created a cone of depression which must have produced a recharge effect from the lake to the main aquifer, even at a very small rate. No recharge effect was noticeable in the water level observation wells, but it definitely did show up in the pumping well. Well 13 reached equilibrium condition after 18 hours of pumping and its drawdown remained at 3.33 feet for the next six hours of pumping. This equilibrium condition had to be produced undoubtedly by a certain amount of recharge.

On the other hand, we found that the top of the aquifer at the site of well 13 and at the site of its satellite wells was between 30 and 31 feet deep. The overlying stratum, which extended up, practically to the ground level, was formed by brown-red clay. Static water level was approximately 15 feet from the ground. This fact indicates that the aquifer was under some artesian or at least semi-artesian conditions. The storage coefficient obtained from the pumping well, approximately 0.013, seems to describe better those conditions than that of approximately 0.080 obtained from the satellite wells.

The drawdown observations from the pumping well which gave sound values for the transmissibility and the storage coefficients were those taken between two and five minutes after pumping had started. Therefore, we have used a flow rate of 5050 gpm in the computations with drawdown data from the pumping well because that was the flow rate which the well was actually discharging for the first seven hours of pumping. The variation of that flow rate with the average of 4980 gpm was, however, less than two per cent. Flow rates were within plus or minus two per cent of the average rate during the entire period of pumping.

Table VI summarizes all the values obtained for the transmissibility and the storage coefficient in accordance with the method employed and the well where water-level observations were made.

TABLE VI

ACUEDUCTO DE MARIANAO, CUBA
PUMPING TEST OF WELL 13
SUMMARY OF RESULTS OBTAINED FOR THE TRANSMISSIBILITY AND THE STORAGE COEFFICIENT WITH DIFFERENT METHODS

Method	Well Used for Water-Level Observations	Distance to Pumping Well feet	Transmissibility T mgd foot	Storage Coefi. S
Jaeger	Pumping Well	1.25	2.63	0.013
Cooper & Jacob	Pumping Well	1.25	2.34	0.014
Theis	Satellite N-1	656	2.33	0.039
Cooper & Jacob	Satellite N-1	656	2.07	0.049
Theis	Satellite N-2	820	2.01	0.110
Cooper & Jacob	Satellite N-2	820	2.03	0.105
Theis	Satellite N-3	986	2.00	0.081
Cooper & Jacob	Satellite N-3	986	2.08	0.071
Jaeger	Satellite N-3	986	1.93	0.080

B. Pumping Test of Well 2, Cia. Cervecera de las Antillas, La Habana, Cuba

By the end of 1958 two wells were developed as the source of industrial water supply of the new brewery Cia. Cervecera de las Antillas, located in the province of La Habana about 12 miles southeast of the city of Havana. Water requirements called for two wells with a 1000 gallons per minute yield from each one. The site for the brewery was selected by a consideration of the water facilities; therefore the two wells were drilled within the property bought for the plant location.

The two wells, drilled 20 inches in diameter, penetrate the entire thickness of an aquifer within the Catalina basin. The aquifer is formed by cavernous limestone geologically classified as from the lower Miocene Epoch. This aquifer is completely independant of the Ariguanabo basin, and at the site of the brewery it has a thickness of about 240 feet. It extends from 60 to 300 feet in depth. Drilling logs indicated slight artesian conditions, although this aquifer is generally regarded as under water-table conditions.

Both wells were tested at about the same pumping rate and with very similar results. The data of well 2 have been selected for this work because during the test of the other well the pumping rate varied excessively due to some difficulties with the pumping equipment.

Well 2 was pumped for 31 consecutive hours at an average rate of 1360 gallons per minute, and water levels were recorded at an observation well located 502 feet from it. The pumping rate from this well varied from 1815 gpm at the very beginning of the test to 1560 gpm at the first hour of pumping, to 1480 gpm at the second hour, to 1340 gpm at the eleventh hour and from then on decreased very slowly to 1280 gpm at the end of the test. The variations were due to deficiencies in the pumping equipment which could not be overcome, however, they did not seem to affect appreciably water-level observations in the satellite well, and it was possible, with such data, to determine the constants of the aquifer.

Table VII shows a summary of the values obtained for the aquifer constants by the writer (Garcia Bengochea, 1960, p. 58) using water-level observations in the satellite well. All these values were computed on the basis of a flow rate of 1280 gpm because the drawdowns recorded for the last part were the ones which could be used for the determinations.

An attempt was made to determine the constants of the aquifer with drawdown data from the pumping well. The Jaeger method was used first. For this purpose, the natural-drawdown graph of the pumping well was drawn as shown on Figure 15. The flow rates recorded during the test are also shown in Figure 15. The reduction in the flow-rate from the pump produced an equilibrium in the pumping well shortly after two hours of pumping, therefore, only those observations made during the first 145 minutes of test are shown on Figure 15.

Computations for T and S were made with the Jaeger method and water level observations from the pumping well as collected in the field, that is, without using the drawdown graph on Figure 15. For each computation we used a value of the flow rate, Q, equal to the

TABLE VII

CERVECERA DE LAS ANTILLAS, SAN JOSE DE LAS LAJAS, CUBA
PUMPING TEST OF WELL 2
VALUES OBTAINED FOR THE TRANSMISSIBILITY AND THE STORAGE COEFFICIENT USING WATER-LEVEL OBSERVATIONS IN SATELLITE WELL

(Garcia Bengochea, 1960, p. 58)

Method	Distance to Pumping Well feet	Transmissibility T gpd/foot	Storage Coefficient S
Theis	502	115,000	0.018
Cooper and Jacob	502	134,000	0.018
Jaeger	502	111,000	0.023

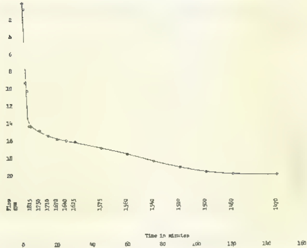


Fig. 15.-Nazura, Time-Drawdown Graph, Carrecera Antilos, Well 2, Drawdowns in Pumping Well

average pumping rate for the corresponding time interval. Such computations are tabulated in Table VIII. An analysis of Table VIII clearly indicates that only those computations corresponding to 10.20 - and 25:50 - minutes time intervals result in values for the storage coefficient which may have any significance in accordance with the geology of the area. These two values of S are respectively 0.013 and 0.037 and the corresponding values for T are 137,000 and 128,000 gpd/foot (gallons per day per foot).

Because only two values were determined for each constant and they seemed to appear in an erratic form, it was decided to make computations between the 10:20 - and the 30.60 - minutes time intervals as shown in Table IX. Values for the drawdown every minute starting at ten minutes were determined from the natural-drawdown graph of Figure 15.

Due to the adjustments of the natural-drawdown graph, the set of observations for 10:20 minutes does not give as good a value for the storage coefficient as the same set in Table VIII. However, it can be observed that those computations between the sets of drawdowns corresponding to 17:34 - and 28:56 - minutes give values of storage coefficient in accordance with the conditions of the aquifer assumed from the driller's logs, that is, ranging from 0.0015 (slightly artesian) to 0.088 (water-table). The corresponding values for T range from 162,000 to 105,000 gpd foot. The arithmetic mean for those computations between 17.34 and 28.56 minutes gives a value for the transmissibility of 136,000 gpd/foot and for the storage coefficient of 0.026.

Figure 16 shows the semi-log plot of Cooper and Jacob with the

TABLE VIII

CERVECERA DE LAS ANTILLAS, SAN JOSE DE LAS LAJAS, CUBA
PUMPING TEST OF WELL 2
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
DRAWDOWN DATA IN THE PUMPING WELL

t' min	$\frac{t'}{t_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/ft	S
2	1	9.17					
4	2	10.17	1.109	4.9×10^{-4}	7.04	143,000	0.15
4	1	10.17					
8	2	14.20	1.398	6.6×10^{-2}	2.21	32,100	9.03
5	1	13.00					
10	2	14.67	1.128	1.2×10^{-3}	6.15	84,600	0.53
10	1	14.67					
20	2	15.65	1.067	9.0×10^{-6}	11.04	137,000	0.013
15	1	15.30					
30	2	16.00	1.045	5.8×10^{-8}	16.09	191,000	0.00018
20	1	15.65					
40	2	16.40	1.048	1.6×10^{-7}	15.07	171,000	0.00058
25	1	15.85					
50	2	16.92	1.068	1.1×10^{-5}	10.84	128,000	0.037
30	1	16.00					
60	2	17.42	1.089	1.2×10^{-4}	8.45	88,900	0.34
40	1	16.40					
80	2	18.42	1.123	9.8×10^{-4}	6.35	61,600	2.56
50	1	16.92					
100	2	19.30	1.141	2.2×10^{-3}	5.54	50,700	5.94
60	1	17.42					
120	2	19.65	1.128	1.2×10^{-3}	6.15	54,500	4.17
75	1	18.17					
150	2	19.67	1.082	6.2×10^{-5}	9.11	79,700	0.39

Q = variable 1515 to 1470 gpm $\quad$ d = 20 inches $\quad$ 4 = 0.835 foot

$$T = \frac{114.6\ Q}{s} W(u)$$

$$S = \frac{u\ T\ t(\text{days})}{1.87\ r^2} = \frac{u\ T\ t'}{1.87\ (0.835)^2 \times 1440} = \frac{u\ T\ t'}{1.88} \times 10^{-3}$$

TABLE IX

CERVECERA DE LAS ANTILLAS, SAN JOSE DE LAS LAJAS, CUBA
PUMPING TEST OF WELL 2
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
INTERPOLATED DRAWDOWN DATA IN THE PUMPING WELL

t' min	$\frac{t'}{t_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
10	1	14.8					
20	2	15.6	1.053	6.3×10^{-7}	13.70	170,000	0.0011
11	1	15.0					
22	2	15.7	1.047	1.1×10^{-7}	15.45	191,000	0.00025
12	1	15.1					
24	2	15.8	1.047	1.1×10^{-7}	15.45	190,000	0.00027
13	1	15.2					
26	2	15.9	1.046	8.0×10^{-8}	15.76	192,000	0.00021
14	1	15.2					
28	2	16.0	1.052	5.0×10^{-7}	13.93	169,000	0.0013
15	1	15.3					
30	2	16.0	1.046	8.0×10^{-8}	15.76	185,000	0.00024
16	1	15.4					
32	2	16.1	1.046	8.0×10^{-8}	15.76	184,000	0.00025
17	1	15.4					
34	2	16.2	1.052	5.0×10^{-7}	13.93	162,000	0.0015
18	1	15.5					
36	2	16.3	1.051	3.8×10^{-7}	14.21	164,000	0.0012
19	1	15.6					
38	2	16.4	1.051	3.8×10^{-7}	14.21	163,000	0.0013
20	1	15.6					
40	2	16.5	1.058	1.9×10^{-6}	12.60	142,000	0.0058
21	1	15.7					
42	2	16.5	1.051	3.8×10^{-7}	14.21	160,000	0.0014
22	1	15.7					
44	2	16.6	1.058	1.9×10^{-6}	12.60	141,000	0.0063
23	1	15.8					
46	2	16.8	1.063	4.7×10^{-6}	11.69	129,000	0.015
24	1	15.8					
48	2	16.8	1.063	4.7×10^{-6}	11.69	129,000	0.015
25	1	15.8					
50	2	16.9	1.070	1.5×10^{-5}	10.53	114,000	0.045
26	1	15.9					
52	2	17.0	1.069	1.2×10^{-5}	10.75	116,000	0.039

TABLE IX - Continued

t' min	$\frac{t'}{t_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
27	1	15.9	1.076	3.2×10^{-5}	9.77	105,000	0.097
54	2	17.1					
28	1	16.0	1.075	2.8×10^{-5}	9.91	105,000	0.088
56	2	17.2					
29	1	16.0	1.082	6.2×10^{-5}	9.11	96,400	0.184
58	2	17.3					
30	1	16.0	1.088	1.1×10^{-4}	8.54	89,700	0.315
60	2	17.4					

Q = variable 1815 to 1470 gpd d = 20 inches r = 0.835 foot

$$T = \frac{114.6\ Q}{s} W(u)$$

$$S = \frac{u\ T\ t\ (\text{days})}{1.87\ r^2} = \frac{u\ T\ t'}{1.87 \times (0.835)^2} = \frac{u\ T\ t'}{1.88} \times 10^{-3}$$

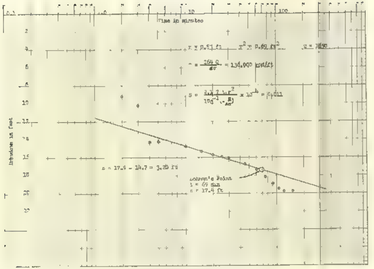


Fig. 16.--Semilog Time-Drawdown Graph, Cervecera Antillas, Wel. [illegible] Drawdown in Pumping Well

drawdown data from the pumping well and the computations to determine T and S with this method and Lohman's modification. T is determined as 136,000 gpd/foot and S as 0.011. Both values were determined with a flow rate of 1650 gpm because this is approximately the average discharge from the pumping well between 10 and 60 minutes of pumping. This period of time corresponds to those observations falling in the straight line.

It is significant to point out that approximately one mile southwest of well 2 there was a boundary in the aquifer formed by older and rather impermeable limestones of the Oligocene Age. Boundary conditions seemed to show in those observations made in the pumping well between 75 and 120 minutes of pumping. This phenomenon also appears in the Jaeger computations of Table VIII where, between the sets of observation 30:60 to 50:100, values of T decrease sharply, while values of S increase even beyond the logical figure of unity. This phenomenon disappeared after 120 minutes of pumping when there was a trend towards stabilization, produced either by the slow reduction in the pumping rate or by a capture in the aquifer or most probably by both.

The apparent boundary effect was not shown in the determination of aquifer constants with drawdown data in the observation well at 502 feet from the pumping well. Figures 17 and 18 (Garcia Bengochea, 1960, pp. 51-56) show respectively the values obtained for T and S with the Jaeger method and drawdowns in the satellite well, and the time-drawdown graph of Cooper and Jacob for drawdowns also in the satellite well. No appreciable deviation seems to take place in either of these two

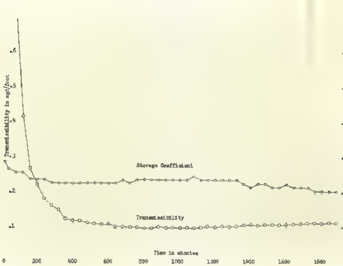


Fig. 17. Values Obtained for T and S, Cervecera Antillas, Well 2, Drawdowns in Satellite Well

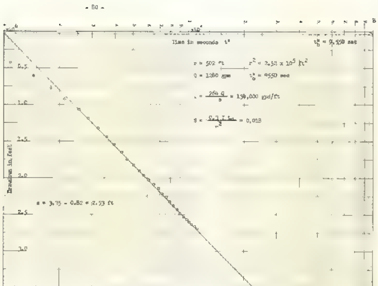


Fig. 18.–Semilog Time-Drawdown Graph, Carrecera Antillas, Well 2, Drawdowns in Satellite Well

graphs after approximately 400 minutes of pumping, a fact which would indicate the existence of that boundary effect.

Table X gives a summary of the results obtained with the different methods and with drawdown data from different wells.

TABLE X

CERVECERA DE LAS ANTILLAS, SAN JOSE DE LAS LAJAS, CUBA
PUMPING TEST OF WELL 2
SUMMARY OF RESULTS OBTAINED FOR THE TRANSMISSIBILITY
AND THE STORAGE COEFFICIENT WITH DIFFERENT METHODS

Method	Well Used for Water-Level Observations	Distance to Pumping Well feet	Transmissibility T gpd/foot	Storage Coeff. S
Jaeger	Pumping well	0.84	136,000	0.026
Cooper & Jacob	Pumping well	0.84	136,000	0.011
Theis	Satellite well	502	115,000	0.018
Cooper & Jacob	Satellite well	502	134,000	0.018
Jaeger	Satellite well	502	111,000	0.023

X. PUMPING TEST AT FORT PIERCE, FLORIDA

Three pumping tests were performed during the summer of 1962 as a part of comprehensive water-supply studies for the City of Fort Pierce, Saint Lucie County, Florida. In order to evaluate the possibilities of a source of ground-water supply test wells were drilled and tested in the northwest section of the city.

The geology of Saint Lucie County indicates that the county is essentially mantled by sands of the Pleistocene age (Cooke, 1945, pl. 1). In the vicinities of Fort Pierce, sand beds extend down to approximately 60 feet in depth and include layers of hardpan, usually near the surface. The lower strata of this sand formation contain small shell, marl, and some clay. Older strata, from the Upper Miocene (Vernon and Babcock, 1962), formed by coarse shell, shell and sand, and consolidated sand and shell, extend between 70 and 130 feet in depth. These older strata constitute a good permeable aquifer under low-pressure artesian conditions. Water from this aquifer, although relatively hard and sometimes slightly colored, is fresh and can be used as a source of water supply (Parker, Ferguson, Love and others, 1955, p. 176). From 120 feet to approximately 700 feet extend strata of marl and other argillaceous materials which serve as the confining beds to the cavernous limestones of the rich Floridan aquifer (Parker, Ferguson, Love and others, 1955, p. 188). Unfortunately, in this area the Floridan aquifer contains highly mineralized water and cannot be used economically as a source of drinking water supply.

Test sites were located on the northwest section of the town, and one four-inch test well was drilled first at each one of the selected sites. These four-inch wells were cased down to the top of the shell, and shell and sand strata, approximately 70 feet deep. Screens of the continuous-slot type, four inches in diameter and ten feet in length were provided at the bottom of the casing, between 70 and 80 feet in depth. Formation samples were collected every five feet and also at every change in the formations penetrated. When the four-inch wells were finished and developed, they were test-pumped with a small deep well turbine pump to obtain information on well production and to collect water samples for water-quality determinations.

Upon completion of the preliminary testing on the four-inch wells, an eight-inch well was drilled at each test site, 100 feet away from each four-inch well. Formation samples were collected also from the eight-inch wells. Samples obtained throughout the entire thickness of the main aquifer, that is, from 70 to 130 feet in depth, were used by the screen manufacturer to determine the slot size of the screens. The eight-inch wells were then cased and provided with 60 feet of eight-inch screen at the bottom.

The purpose of the screens in the wells is to prevent the unconsolidated formations of shell and sand from caving in and to allow the water in the formation to flow through it with a minimum loss of head. After each screen was set, the well was then developed. The purpose of the development was to flush from the formation surrounding the screen all finer particles of sand and shell, as well as marl and other materials which might resist the flow of water into the well. This was accomplished by jetting horizontally the formation around the screen with high velocity water jets from a tool inside the screen, and further, by surging the well with compressed air. The selection of the proper screen and slot size, together with the development work, are the most important factors in obtaining a successful screened well drilled in a suitable formation (Bennison, 1947, p. 219).

After the development work on the eight-inch wells was completed, a pumping test was conducted in each well. Pumping tests were performed for several hours on each eight-inch well while recording the rate of flow and the water level in both the pumping well and the four-inch satellite well. Rates of flow were determined by recording the piezometric head on an orifice plate at the end of the discharge pipe. Water levels in the pumping well were measured by means of the nitrogen bubbler connected to a mercury piezometer and recorder. Water levels at the four-inch wells were recorded with a Stevens and Leupold, type F, water level recorder.

Preliminary pumping tests were made in each eight-inch well before final testing. This was done to predetermine an appropriate pumping rate and to check instruments and equipment. Sufficient time elapsed between the tests to assure complete recovery. Figure 19 shows a view of the pumping test at well 10A.

Eight-inch wells were designated 9A, 10A and 11A, corresponding respectively to the four-inch wells designated 9, 10 and 11 and drilled prior to the drilling of the larger wells. In the performance of the first two tests, at wells 9A and 11A, field conditions prevented the

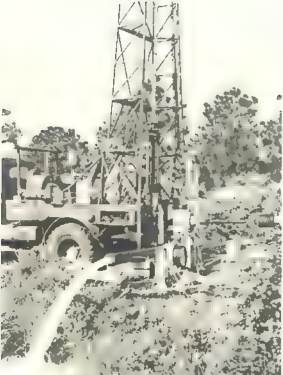


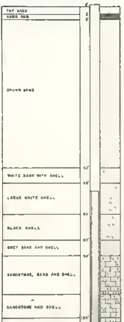
Fig. 9.-Pumping test of well 10A, City of Fort Pierce

writer from making water-level observations in the pumping well with the bubbler system as frequently and as accurately as he had intended. Therefore, data from those two tests have not been used in this evaluation.

A summary of the field data obtained from well 10A, including driller's log and water analysis is shown on Figure 20. This figure also gives the exact position of the bottom of the casing and the extent of the screen. An analysis of the well log and information on the static water level, 12.5 feet from top of casing (July 11, 1962), together with the fact that the permeability of the overlying sands is considerably less than that of the shell aquifer, lead us to the assumption that the shell aquifer should be under low-pressure artesian conditions.

Well 10A was pumped at the constant rate of flow of 400 gpm and the water level stabilized in the satellite well, after only 85 minutes of pumping, with a maximum drawdown of 3.07 feet. The water level in the pumping well reached equilibrium after approximately 60 minutes of pumping, with a maximum drawdown of 29.21 feet.

Figure 21 shows the natural time-drawdown graph of water-level observations in the pumping well for the first one thousand seconds of pumping. Water-level readings were made in this well starting at every five seconds at the beginning of the test. Figure 21 also shows the residual drawdowns for the first one thousand seconds immediately after the pump was shut down. Maximum drawdown minus residual drawdown gives the recovery. Recovery data have been used also in the determination of aquifer constants.



WELL INFORMATION

WELL CITY OF FORT PIERCE
TEST WELL NO 10A

LOCATION: 29th STREET AND AVENUE Q ON WEST SIDE OF CANAL RIGHT OF-WAY 100' SOUTH OF AVENUE Q.

COUNTY: ST LUCIE **STATE** FLORIDA

CASING ELEVATION: 16.83 FEET M S L

DIAMETER 8 INCHES

DEPTH: 130 FEET

STATIC WATER LEVEL 12' 6" (+3.52')
DATE JULY 11, 1962

CASING 71' 8" BLACK STEEL THREADS AND COUPLINGS

SCREEN: 20' OF JOHNSON ARMCO IRON 6" Ø SLOT NO 60- 71 80' SLOT NO. 60- 81 111' SLOT NO 40- 111 131 TELESCOPE TYPE

CONSTRUCTION: ROTARY

DRILLER SOUTHERN DRILLERS, INC
JACKSONVILLE, FLORIDA

DATE FINISHED: JULY 12, 1962

SPECIFIC YIELD: 25 G.P.M PER FOOT

WATER ANALYSIS

AFTER 30 HOURS OF PUMPING AT 400 G.P.M.

T.D.S	388	ppm
SiO_2	14	ppm
IRON AS Fe	0.24	ppm
CALCIUM AS $CaCO_3$	172	ppm
MAGNESIUM AS $CaCO_3$	28	ppm
METHYL ORANGE ALKALINITY AS $CaCO_3$	168	ppm
TOTAL HARDNESS AS $CaCO_3$	200	ppm
$SO_4^{=}$	8.5	ppm
Cl^-	53	ppm
ORGANIC COLOR	50	
pH	7.60	

Fig. 20.-Summary of Well Information, City of Fort Pierce, Well 10A

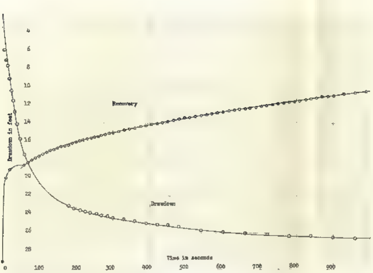


Fig. 21.-Natural Time-Drawdown Graph, City of Fort Pierce, Well 10A, Drawdowns in Pumping Well

Jaeger method computations for different time intervals were made starting with drawdown observations in the pumping well every five seconds. These computations are shown on Table XI. Values obtained from this table for the transmissibility and the storage coefficient were plotted versus the corresponding time and are shown in Figure 22.

An analysis of Table XI and Figure 22 indicates that up to the 210:420 second intervals the values obtained for the storage coefficient do not correspond to the low-pressure artesian conditions found from the drilling logs and the geology of the area. This analysis also indicates that after the 210:420-second interval the values obtained for T and S varied very little until the 330:660-second interval when the transmissibility started to show a more marked rate of increase, and the storage coefficient began to decline somewhat more rapidly. Both conditions are always associated with recharge effects. Because of the small time intervals used in these computations, the changes of the values for T and S, shown in Table XI and on Figure 22 are also small.

The arithmetic mean for the values of T between 210:440 seconds and 330:660 seconds is 20,300 gpd/foot and that for S is 0.0032. These two figures will be considered the values obtained for T and S with the Jaeger method and drawdown data from the pumping well.

Table XII gives the computations and values obtained for T and S with recovery data from the pumping well. The recovery data were determined by subtracting the residual drawdowns, given in Figure 22, from the maximum stabilized drawdown during the test. It is important to indicate the break in the residual drawdown curve between 20 and 60

TABLE XI

CITY OF FORT PIERCE, FLORIDA
PUMPING TEST OF TEST WELL 10A
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
DRAWDOWN DATA IN PUMPING WELL

t" sec	$\frac{t''}{t''_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/ft	S
5	1	5.92					
10	2	7.10	1.198	9.8×10^{-3}	4.06	26,200	1.4×10^{-1}
10	1	7.10					
20	2	9.18	1.292	3.1×10^{-2}	2.93	14,600	5.1×10^{-1}
15	1	7.71					
30	2	11.56	1.50	1.1×10^{-1}	1.74	6,900	1.3×10^{0}
20	1	9.18					
40	2	14.18	1.54	1.3×10^{-1}	1.59	5,000	1.5×10^{0}
25	1	10.53					
50	2	15.78	1.50	1.1×10^{-1}	1.74	5,100	1.6×10^{0}
30	1	11.56					
60	2	17.42	1.51	1.1×10^{-1}	1.74	4,600	1.7×10^{0}
35	1	12.70					
70	2	18.77	1.478	1.0×10^{-1}	1.82	4,400	1.7×10^{0}
40	1	14.18					
80	2	19.91	1.406	6.8×10^{-2}	2.18	5,000	1.5×10^{0}
45	1	14.88					
90	2	20.53	1.389	6.2×10^{-2}	2.26	5,100	1.6×10^{0}
50	1	15.78					
100	2	21.01	1.332	4.4×10^{-2}	2.59	5,700	1.4×10^{0}
55	1	16.50					
110	2	21.39	1.296	3.2×10^{-2}	2.90	6,200	1.2×10^{0}
60	1	17.42					
120	2	21.71	1.245	1.9×10^{-2}	3.40	7,000	8.9×10^{-1}
65	1	18.01					
130	2	22.00	1.222	1.4×10^{-2}	3.71	7,700	7.8×10^{-1}
70	1	18.77					
140	2	22.28	1.188	7.2×10^{-3}	4.36	9,000	5.1×10^{-1}
75	1	19.44					
150	2	22.50	1.158	3.6×10^{-3}	5.05	10,300	3.1×10^{-1}
80	1	19.91					
160	2	22.70	1.140	2.1×10^{-3}	5.59	11,300	2.1×10^{-1}
85	1	20.25					
170	2	22.89	1.131	1.4×10^{-3}	6.00	12,000	1.6×10^{-1}
90	1	20.53					
180	2	23.07	1.125	1.0×10^{-3}	6.33	12,600	1.3×10^{-1}

TABLE XI - Continued

t" sec	$\frac{t''}{t''_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/ft	S
95	1	20.80					
190	2	23.20	1.116	7.2×10^{-4}	6.66	13,200	1.0×10^{-1}
100	1	21.01					
200	2	23.32	1.109	5.0×10^{-4}	7.02	13,800	7.7×10^{-2}
110	1	21.39					
220	2	23.58	1.102	3.3×10^{-4}	7.44	14,200	5.6×10^{-2}
120	1	21.71					
240	2	23.81	1.097	2.3×10^{-4}	7.80	14,700	4.3×10^{-2}
130	1	22.00					
260	2	24.01	1.092	1.5×10^{-4}	8.23	15,400	3.4×10^{-2}
140	1	22.28					
280	2	24.19	1.086	9.2×10^{-5}	8.72	16,200	1.2×10^{-2}
150	1	22.50					
300	2	24.34	1.082	6.2×10^{-5}	9.11	16,800	1.8×10^{-2}
160	1	22.70					
320	2	24.49	1.078	4.1×10^{-5}	9.52	17,500	1.3×10^{-2}
170	1	22.89					
340	2	24.61	1.076	3.4×10^{-5}	9.71	17,700	1.1×10^{-2}
180	1	23.07					
360	2	24.72	1.072	1.9×10^{-5}	10.29	18,700	7.2×10^{-3}
190	1	23.70					
380	2	24.84	1.071	1.7×10^{-5}	10.41	18,800	6.8×10^{-3}
200	1	23.32					
400	2	24.97	1.070	1.5×10^{-5}	10.53	18,900	6.4×10^{-3}
210	1	23.48					
420	2	25.06	1.067	9.0×10^{-6}	11.04	19,800	4.0×10^{-3}
220	1	23.58					
440	2	25.18	1.067	9.0×10^{-6}	11.04	19,700	4.4×10^{-3}
230	1	23.70					
460	2	25.28	1.067	9.0×10^{-6}	11.04	19,600	4.4×10^{-3}
240	1	23.81					
480	2	25.36	1.065	6.6×10^{-6}	11.35	20,100	3.5×10^{-3}
250	1	23.91					
500	2	25.47	1.065	6.6×10^{-6}	11.35	20,100	3.6×10^{-3}
260	1	24.01					
520	2	25.54	1.063	4.7×10^{-6}	11.69	20,600	2.8×10^{-3}
270	1	24.10					
540	2	25.62	1.063	4.7×10^{-6}	11.69	20,500	2.9×10^{-3}
280	1	24.19					
560	2	25.70	1.063	4.7×10^{-6}	11.69	20,400	3.0×10^{-3}

TABLE XI - Continued

t'' sec	$\frac{t''}{t''_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/ft	S
290	1	24.27					
580	2	25.78	1.062	3.9×10^{-6}	11.88	20,700	2.6×10^{-3}
300	1	24.34					
600	2	25.85	1.062	3.9×10^{-6}	11.88	20,600	2.6×10^{-3}
310	1	24.42					
620	2	25.91	1.061	3.2×10^{-6}	12.08	20,900	2.3×10^{-3}
320	1	24.49					
640	2	25.98	1.061	3.2×10^{-6}	12.08	20,900	2.4×10^{-3}
330	1	24.59					
660	2	26.01	1.058	1.9×10^{-6}	12.60	21,800	1.5×10^{-3}
340	1	24.61					
680	2	26.08	1.059	2.2×10^{-6}	12.45	21,400	1.8×10^{-3}
350	1	24.67					
700	2	26.12	1.059	2.2×10^{-6}	12.45	21,400	1.8×10^{-3}
360	1	24.72					
720	2	26.17	1.058	1.9×10^{-6}	12.60	21,600	1.6×10^{-3}
370	1	24.79					
740	2	26.21	1.057	1.5×10^{-6}	12.83	22,000	1.4×10^{-3}
380	1	24.84					
760	2	26.24	1.056	1.2×10^{-6}	13.06	22,300	1.1×10^{-3}
390	1	24.90					
780	2	26.28	1.055	1.0×10^{-6}	13.24	22,600	9.6×10^{-4}
400	1	24.97					
800	2	26.31	1.054	8.0×10^{-7}	13.46	23,000	8.0×10^{-4}
410	1	25.01					
820	2	26.33	1.053	6.2×10^{-7}	13.72	23,400	6.4×10^{-4}
420	1	25.06					
840	2	26.38	1.053	6.2×10^{-7}	13.72	23,400	5.2×10^{-4}
430	1	25.12					
860	2	26.41	1.051	4.8×10^{-7}	13.97	23,700	5.2×10^{-4}
440	1	25.18					
880	2	26.44	1.051	4.8×10^{-7}	13.97	23,700	5.6×10^{-4}
450	1	25.22					
900	2	26.48	1.049	2.2×10^{-7}	14.75	25,000	2.7×10^{-4}
460	1	25.28					
920	2	26.51	1.048	1.7×10^{-7}	15.01	25,400	2.2×10^{-4}
470	1	25.32					
940	2	26.54	1.048	1.7×10^{-7}	15.01	25,400	2.2×10^{-4}
480	1	25.36					
960	2	26.58	1.048	1.7×10^{-7}	15.01	25,400	2.3×10^{-4}

TABLE XI - Continued

t'' sec	$\frac{t''}{t''_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/ft	S
490	1	25.41					
980	2	26.61	1.047	1.1×10^{-7}	15.45	25,900	1.6×10^{-4}
500	1	25.47					
1000	2	26.63	1.046	8.0×10^{-8}	15.76	26,600	1.2×10^{-4}

Q = 400 gpm　　d = 8 inches　　r = 0.33 foot

$$T = \frac{114.6\ Q}{s} W(u) = \frac{114.6 \times 400}{s} W(u) = 44{,}900 \frac{W(u)}{s}$$

$$S = \frac{u\ T\ t}{1.87\ r^2} = \frac{u\ T\ t''}{1.87 \times (0.33)^2 \times 86{,}400} = \frac{u\ T\ t''}{1.79} \times 10^{-4}$$

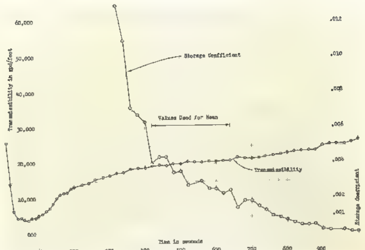


Fig. 22.-Values Obtained for T and S, City of Fort Pierce, Well 10A, Drawdowns in Pumping Well

TABLE XII

CITY OF FORT PIERCE, FLORIDA
PUMPING TEST OF TEST WELL 10A
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
RECOVERY DATA IN THE PUMPING WELL

t'' sec	$\frac{t''}{t''_o}$	s_r feet	$\frac{s}{s_b}$	u	W(u)	T gpd/ft	β
10	1	9.21					
20	2	10.12	1.101	3.2×10^{-4}	7.47	33,100	1.2×10^{-2}
Observations affected by deep-well pump backflow							
60	1	10.63					
120	2	11.99	1.128	1.2×10^{-3}	6.15	23,000	1.8×10^{-1}
70	1	10.91					
140	2	12.33	1.131	1.4×10^{-3}	6.00	21,800	2.3×10^{-1}
80	1	11.19					
160	2	12.60	1.127	1.2×10^{-3}	6.15	21,900	2.3×10^{-1}
90	1	11.39					
180	2	12.83	1.127	1.2×10^{-3}	6.15	21,500	2.6×10^{-1}
100	1	11.60					
200	2	13.13	1.132	1.5×10^{-3}	5.93	20,300	3.4×10^{-1}
110	1	11.80					
220	2	13.38	1.133	1.6×10^{-3}	5.86	19,700	3.9×10^{-1}
120	1	11.99					
240	2	13.59	1.133	1.6×10^{-3}	5.86	19,400	4.2×10^{-1}
130	1	12.17					
260	2	13.81	1.135	1.7×10^{-3}	5.80	18,800	4.5×10^{-1}
140	1	12.33					
280	2	14.03	1.138	1.8×10^{-3}	5.74	18,400	5.2×10^{-1}
150	1	12.48					
300	2	14.21	1.139	1.9×10^{-3}	5.69	18,000	5.7×10^{-1}
160	1	12.60					
320	2	14.40	1.142	2.2×10^{-3}	5.54	17,300	6.8×10^{-1}
170	1	12.72					
340	2	14.57	1.143	2.3×10^{-3}	5.50	16,900	7.4×10^{-1}
180	1	12.83					
360	2	14.72	1.148	2.7×10^{-3}	5.34	16,600	9.0×10^{-1}
190	1	13.01					
380	2	14.90	1.145	2.5×10^{-3}	5.42	16,300	8.7×10^{-1}
200	1	13.13					
400	2	15.08	1.148	2.7×10^{-3}	5.34	15,900	9.6×10^{-1}

TABLE XII - Continued

t'' sec	$\frac{t''}{t''_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/ft	S
300	1	14.21					
600	2	16.51	1.162	6.4×10^{-3}	4.48	17,200	2.6×10^{0}
400	1	15.08					
800	2	17.69	1.163	6.5×10^{-3}	4.47	11,300	3.3×10^{0}
500	1	15.85					
1000	2	18.77	1.183	7.9×10^{-3}	4.27	10,200	4.4×10^{0}

Q = 400 gpm d = 8 inches r = 0.33 ft

$$T = \frac{114.6\ Q}{s_r} W(u) = \frac{114.6 \times 400}{s_r} = 44,900 \frac{W(u)}{s_r}$$

$$S = \frac{u\ T\ t_r}{1.87\ r^2} \equiv \frac{u\ T\ t''_r}{1.87 \times (0.33)^2 \times 86,400} = \frac{u\ T\ t''}{1.79} \times 10^{-4}$$

seconds after pumping stopped shown on Figure 22. This break was almost certainly due to the backflow to the well of all the water contained in the deep-well pump and in the pump column immediately after pumping was stopped. The inertia of the pumping equipment may also alter these first observations, however, in this particular instance the variations seem to be characteristic of backflow conditions. Because of the break on the residual-drawdown curve, no computations are included in Table XII between the 10:20- and the 60.120-second intervals.

The values obtained in Table XII for the storage coefficient do not resemble at all the low-pressure artesian conditions of the aquifer, although the values of the transmissibility for the first minutes of recovery agree with those calculated with the drawdown data. This phenomenon seems to be produced by the backflow of water to the well; however, the fact is that the values obtained for the storage coefficient are not acceptable; consequently this method using such recovery data does not offer enough reliability.

Figure 23 shows the semilog time-drawdown plot of Cooper and Jacob and the computations for T and S with drawdown data from the pumping well. Results shown on Figure 23 are as follows:

$T = 21,100$ gpd/foot

$S = 0.0025$

The rapid stabilization of the water level in both the pumping and the satellite well, together with geological considerations, indicated the presence of recharge effects due to leaky aquifer conditions. Therefore, it was decided to use the method of the "Type Curves for

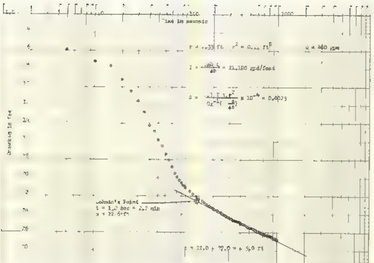


Fig. [illegible] Semilog Time-Drawdown Graph, City of Fort Pierce, Well [illegible], Drawdowns in Pumping Well

Nonsteady Radial Flow in an Infinite Leaky Aquifer" developed by Cooper (1963) and described in Chapter VII. This method as indicated by its developer constitutes an extension of Theis method and hence we have used the term "Theis-Cooper Graph" to designate the plot of drawdowns versus time, necessary for the Type-Curves or the Theis-Cooper method.

Figure 24 shows the Theis-Cooper graph with drawdown data from the pumping well together with the computations and results for the aquifer constants. These are:

$T = 5,300$ gpd/foot

$S = 0.034$

$P' = 0.062$ gpd/sq.ft.

So far we have seen that there is a rather close agreement between the results obtained with the Jaeger method and the Cooper and Jacob semilog plot, but that there is a marked discrepancy with the Type-Curves method.

We proceeded then to determine the aquifer constants with the same three methods but with drawdown data from the satellite well.

Figure 25 shows the natural time-drawdown graph from observations in the satellite well. As stated before, the water level in this well stabilized after 85 minutes of pumping with a maximum drawdown of 3.07 feet. Three minutes was the smallest time interval which could be read from the chart of the water-level recorder installed in this well, however, with the natural time-drawdown graph we can interpolate readings for shorter time intervals.

Computations with the Jaeger method have been made, starting with one-half minute time intervals, and are shown on Table XIII. This table

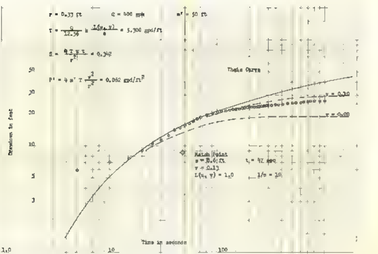


Fig. 24.-Theis-Cooper Graph, City of Fort Pierce, Well 10A, Drawdowns in Pumping Well

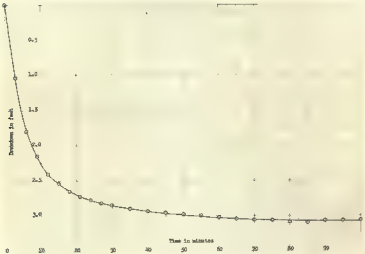


Fig. 25.-Natural Time-Drawdown Graph, City of Fort Pierce, Well 10A, Drawdowns in Satellite Well

TABLE XIII

CITY OF FORT PIERCE, FLORIDA
PUMPING TEST OF TEST WELL 10A
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
DRAWDOWN DATA IN SATELLITE TEST WELL 10

t' min	$\frac{t'}{t_o}$	s feet	$\frac{p}{s_o}$	u	W(u)	T gpd/ft	S
0.5	1	0.16					
1.0	2	0.33	2.06	3.2×10^{-1}	0.858	117,000	1.4×10^{-3}
1.0	1	0.33					
2.0	2	0.70	2.12	3.5×10^{-1}	0.794	51,000	1.3×10^{-3}
1.5	1	0.52					
3.0	2	1.05	2.02	3.1×10^{-1}	0.881	37,700	1.3×10^{-3}
2.0	1	0.70					
4.0	2	1.40	2.00	3.0×10^{-1}	0.906	29,100	1.3×10^{-3}
2.5	1	0.88					
5.0	2	1.64	1.86	2.5×10^{-1}	1.044	28,600	1.3×10^{-3}
3.0	1	1.05					
6.0	2	1.80	1.45	8.6×10^{-2}	1.96	48,800	9.4×10^{-4}
3.5	1	1.24					
7.0	2	1.94	1.56	1.3×10^{-1}	1.59	36,800	1.2×10^{-3}
4.0	1	1.40					
8.0	2	2.05	1.43	7.7×10^{-2}	2.06	45,100	1.0×10^{-3}
4.5	1	1.53					
9.0	2	2.16	1.41	7.0×10^{-2}	2.15	44,700	1.0×10^{-3}
5.0	1	1.64					
10.0	2	2.25	1.37	5.6×10^{-2}	2.36	47,100	9.8×10^{-4}
5.5	1	1.73					
11.0	2	2.33	1.35	4.9×10^{-2}	2.49	48,000	9.6×10^{-4}
6.0	1	1.80					
12.0	2	2.40	1.33	4.3×10^{-2}	2.61	48,800	9.4×10^{-4}
6.5	1	1.87					
13.0	2	2.45	1.31	3.7×10^{-2}	2.76	50,400	9.0×10^{-4}
7.0	1	1.94					
14.0	2	2.49	1.28	2.8×10^{-2}	3.03	54,600	8.0×10^{-4}
7.5	1	2.00					
15.0	2	2.54	1.27	2.5×10^{-2}	3.14	55,500	7.7×10^{-4}
8.0	1	2.05					
16.0	2	2.58	1.26	2.3×10^{-2}	3.22	56,000	7.7×10^{-4}
8.5	1	2.11					
17.0	2	2.62	1.24	1.8×10^{-2}	3.46	59,300	6.7×10^{-4}

TABLE XIII - Continued

t' min	$\frac{t'}{t_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/ft	S
9.0	1	2.16					
18.0	2	2.65	1.23	1.5×10^{-2}	3.64	61,700	6.2×10^{-4}
9.5	1	2.21					
19.0	2	2.68	1.21	1.1×10^{-2}	3.94	66,000	5.1×10^{-4}
10.0	1	2.25					
20.0	2	2.70	1.20	9.2×10^{-3}	4.12	68,500	4.7×10^{-4}
12.5	1	2.42					
25.0	2	2.80	1.157	3.5×10^{-3}	5.08	81,400	2.6×10^{-4}
15.0	1	2.54					
30.0	2	2.85	1.122	9.5×10^{-4}	6.38	100,500	1.1×10^{-4}
17.5	1	2.63					
35.0	2	2.89	1.099	2.6×10^{-4}	7.68	119,500	4.0×10^{-5}
20.0	1	2.70					
40.0	2	2.93	1.085	7.4×10^{-5}	8.93	137,000	1.5×10^{-5}
22.5	1	2.75					
45.0	2	2.95	1.073	2.2×10^{-5}	10.19	155,000	5.7×10^{-6}
25.0	1	2.80					
50.0	2	2.98	1.064	5.6×10^{-6}	11.52	174,000	1.8×10^{-6}
27.5	1	2.83					
55.0	2	3.00	1.060	2.7×10^{-6}	12.24	186,000	1.0×10^{-6}
30.0	1	2.85					
60.0	2	3.02	1.059	2.3×10^{-6}	12.41	184,500	9.5×10^{-7}
32.5	1	2.87					
65.0	2	3.04	1.059	2.3×10^{-6}	12.41	183,000	9.4×10^{-7}
35.0	1	2.89					
70.0	2	3.05	1.056	1.3×10^{-6}	12. 98	191,000	6.5×10^{-7}
37.5	1	2.91					
75.0	2	3.06	1.052	5.0×10^{-7}	13.93	209,000	2.9×10^{-7}
40.0	1	2.93					
80.0	2	3.07	1.047	1.1×10^{-7}	15.45	226,000	7.4×10^{-8}
42.5	1	2.94					
85.0	2	3.08	1.047	1.1×10^{-7}	15.45	225,000	7.8×10^{-8}
45.0	1	2.95					
90.0	2	3.08	1.044	4.0×10^{-8}	16.46	240,000	3.2×10^{-8}
47.5	1	2.97					
95.0	2	3.08	1.036	1.3×10^{-9}	19.88	290,000	1.3×10^{-9}
50.0	1	2.98					
100.0	2	3.08	1.033	2.2×10^{-10}	21.66	316,000	2.6×10^{-10}

TABLE XIII - Continued

Q = 400 gpm d = 8 inches r = 100 feet

$T = \frac{114.6\ Q}{s}\ W(u) = \frac{114.6 \times 400}{s}\ W(u) = 44{,}900\ \frac{W(u)}{s}$

$S = \frac{u\ T\ t}{1.87\ r^2} = \frac{u\ T\ t'}{1.87 \times 100^2 \times 1{,}440} = \frac{u\ T\ t'}{2.69} \times 10^{-7}$

indicates that the values obtained for S had little change with time for the first minutes of pumping; however, T started high, decreased, tried to stabilize between two to five minutes and then increased rather rapidly in the typical trend of recharge conditions. Table XIII also indicates that there were field conditions which did not permit the straight application of the non-equilibrium formula and therefore of this method's computations except for the 20:40-minutes time intervals. Aware of the low reliability of the figures, we have chosen the arithmetic mean of the results obtained from those two time intervals (2.0:4.0- and 2.5:5.0 minutes) as the most probable values for T and S. These are: T = 28,800 gpd/foot

S = 0.0013

The semilog plot with the drawdown data from this well, shown on Figure 26, also indicates recharge effects. The best adapted straight line we could get from this graph gives the following results.

T = 45,000 gpd/foot

S = 0.0009

The best method to apply under these conditions would be Cooper's Type Curves. Figure 27 shows the corresponding Theis-Cooper graph with results as follows: T = 32,300 gpd/foot

S = 0.0014

P' = 26 gpd/sq.ft.

We will proceed now with the following reasoning:

1. Equilibrium conditions were reached in both the pumping and the satellite well, equilibrium conditions which the geology of the area indicated should have been caused by leakage from the overlying

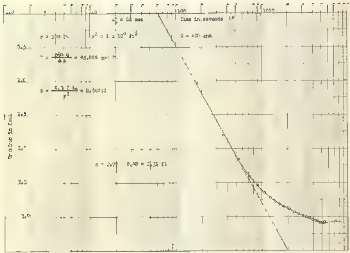


Fig [illegible] -Semilog Time-drawdown Graph, [illegible] of [illegible] , Well 10A, Drawdowns in Satellite Well

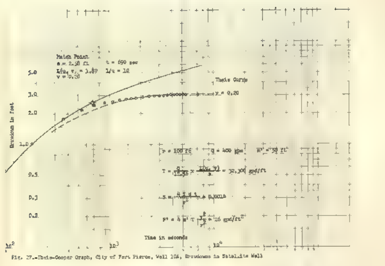


Fig. 27.-Theis-Cooper Graph, City of Fort Pierce, Well 104, Drawdowns in Satellite Well

low-permeable sand beds. Therefore, Jacob's equation (20) described in Chapter VII should be valid. This equation is

$$s_m = \frac{Q}{2\pi T} \; K_0(x) \qquad (20)$$

2. In both wells we know s_m (maximum drawdown under equilibrium conditions).

3. The Jaeger method clearly indicated that after pumping began, there were periods in which the drawdown data deviated from the theory, and then the results obtained for T and S were not valid. But it also indicated that there were other periods, sometimes very short, in which there was an agreement between the theory and the hydraulic behavior of the aquifer. The results obtained for T and S during those last periods, should identify then, the transmissibility and the storage coefficient of the aquifer, characteristics of the aquifer which are essentially constants.[1]

4. Therefore, the values obtained for T and S with the Jaeger and the Cooper and Jacob methods should represent the real value of the aquifer constants, if the right observations have been selected by the investigator.

We know then, s_m and T, and because with pumping we reached equilibrium conditions, we can calculate the vertical permeability P' with the expressions:

---

[1] The storage coefficient may vary if, because of changes in the piezometric head, an aquifer under artesian conditions changes to water-table conditions or vice versa.

$$s_m = \frac{Q}{2\pi T} K_0(x) \qquad (20)$$

$$x = r\sqrt{\frac{P'}{Tm'}} \qquad (17)$$

and with the aid of a table of values of $K_0(x)$, the modified Bessel function of the second kind of zero order, for values of x. The value for m' in this test was determined as 50 feet in accordance with the data on Figure 20.

Table XIV gives a summary of the values obtained for T, S, and P' using the procedures described. An analysis of this table indicates that although there is quite a discrepancy between the results, those obtained with the Jaeger method show a better agreement between the values obtained from the satellite well and from the pumping well.

It is important to point out from Table XIV that, although the Theis-Cooper method is the best adapted to these test conditions, it did not give satisfactory results when applied with drawdown data from the pumping well.

Table XIV also shows a difference between the round figures of 30,000 gpd/foot for T and 0.0013 for S obtained with drawdown data from the satellite well, and those of approximately 21,000 gpd/foot and 0.003 for S obtained with drawdown data from the pumping well. In all our computations with drawdown data from the pumping well, we have substituted in every expression the parameter r, with the actual radius of the well, and also we have neglected any loss of head through the screen. These two factors may explain the difference.

More light could have been thrown on the above difference with an additional multiple-step drawdown test as outlined by Jacob (1947,

TABLE XIV

CITY OF FORT PIERCE, FLORIDA
PUMPING TEST OF TEST WELL 10A
SUMMARY OF RESULTS OBTAINED FOR AQUIFER CONSTANTS
WITH DIFFERENT METHODS USING DRAWDOWN DATA

Method	Well for Water-Level Observations	Trans-missibility T gpd foot	Storage Coefficient S -	Vertical Permeability P' gpd sq.ft.
Jaeger	Pumping	20,300	0.0032	28
Cooper & Jacob	Pumping	21,100	0.0025	20
Theis-Cooper	Pumping	5,300	0.34	0.06
Jaeger	Satellite	28,600	0.0013	30
Cooper & Jacob	Satellite	45,000	0.0009	5
Theis-Cooper	Satellite	32,300	0.0014	26

p. 1047) with the additional considerations by Rorabaugh (1953, 23 pp.). However, we should not forget that the values obtained for the aquifer constants cannot be taken strictly as accurate mathematical numbers, but rather, as coefficients which give an idea of the aquifer and which may also be used to estimate future conditions within reasonable limits.

To give a better explanation of the previous statement, we have calculated maximum drawdowns with the results obtained from two of the above methods, which in our opinion seem to be more reliable: the Theis-Cooper method with drawdown data from the satellite well and the Jaeger method with drawdown data from the pumping well. Drawdowns have been calculated for the pumping well and for points in the aquifer located respectively from the pumping well at: 100 feet (satellite well), 500 feet and 1000 feet. Table XV gives the calculated drawdowns and also the actual maximum drawdowns as measured in the pumping and satellite wells.

Table XV indicates that in this particular aquifer, the maximum drawdowns calculated with aquifer constants properly determined from either the pumping or the satellite well are practically identical for points in the aquifer 500 feet or more from the pumping well. At 100 feet the maximum drawdown calculated with data from the pumping well is 1.1 foot greater than the value directly measured that is, the computed value is on the safe side. Between the pumping well and 100 feet away from it, the calculated values with data from the pumping well are either correct (at the pumping well) or on the safe side. However, the computed values with data from the satellite well are either correct (at the satellite well) or smaller than the actual values, the maximum difference of 9.9 feet being at the pumping well (34 per cent of the measured value).

Most often we are interested in forecasting the drawdowns at the pumping well and the interference with another pumping well, usually situated at least 300 feet or more from the first one. Therefore, we may realize that the values of the aquifer constants determined with data from the pumping well seem to give better forecasts than those values obtained from an observation well.

TABLE XV

CITY OF FORT PIERCE, FLORIDA
PUMPING TEST OF TEST WELL 10A
COMPARISON OF MEASURED AND CALCULATED MAXIMUM DRAWDOWNS AT PUMPING WELL AND VICINITIES USING AQUIFER CONSTANTS DETERMINED FROM THE PUMPING AND SATELLITE WELL

	Maximum Drawdowns in Feet at			
	Pumping	Distance from Pumping Well		
Determination (1)	Well	100 ft.	500 ft.	1000 ft.
Calculated with T and S from				
Theis-Cooper, Satellite Well	19.3	3.2	0.31	0.03
Jaeger, Pumping Well	29.6	4.3	0.32	0.02
Direct Measurement	29.2	3.1	-	-

(1) Flow rate from pumping well at 400 gpm

XI. PUMPING TEST AT LEESBURG, FLORIDA

The City of Leesburg, Lake County, in the central part of the peninsula of Florida, uses ground water for its public water supply. Ground water is secured in Leesburg with deep, open-hole wells, which penetrate into the Floridan aquifer.

The top of the Floridan aquifer is formed in this area by porous and cavernous limestones of the Ocala formation, Upper Eocene age. Below the Ocala limestones, wells penetrate in this area Moodys Branch and the Avon Park formations, both formed by limestones of similar hydrologic and lithologic properties to the Ocala limestones. Overlying the Floridan aquifer and confining its water under artesian pressure are strata of sand with some streaks of clay, of relatively low permeability. These strata vary from Pleistocene on the surface to Miocene just above the Ocala limestones (Jordan, 1952). Ground water from the Floridan aquifer in this area is moderately hard and contains total dissolved solids of approximately 180 ppm (Black and Brown, 1951, p. 70).

At the writing of this paper, three wells 16 inches in diameter are being drilled on the north shore of Lake Harris, within Leesburg city limits. They are located in an east-west straight line, approximately one thousand feet apart, the well in the middle being just by the Leesburg Community Center. Of these wells, number 12 is the easternmost and the first one which has been finished and pump-tested.

Well 12 is 365 feet in total depth and was cased down to 185 feet, the bottom part being open hole. The top of the Floridan aquifer appeared at 181 feet, the overlying beds being sand strata with some streaks of clay of relatively low permeability.

There was no well in the vicinities of well 12 which could be used as an observation or satellite well; therefore, only drawdown observations were made on the pumping well. The drilling of the other two wells had not been started when well 12 was tested.

Two drawdown tests were performed in this well, one before it was completely developed and the water from the well still turbid with silt and sand, and another test after the well was completely developed.

Well 12 was pumped before the first test for 31 1/2 consecutive hours, and the pumping rate was steadily increased from 600 gpm to 1600 gpm. This was done in order to clean the well from sand, well cuttings and mud. At the end of this period the water was not completely clear and still had some sand, however, it was decided to conduct a drawdown test after enough time was allowed for the complete recovery of the water level.

For the first drawdown test, well 12 was pumped at the constant rate of 1220 gpm and drawdown observations were made in it with the nitrogen-bubbler system. A view of the installation is shown on Figure 28.

The water level in pumping well 12 stabilized after 160 minutes of pumping with a maximum drawdown of 38.2 feet. The increase in drawdown from the observation at 90 minutes of pumping to the maximum drawdown was only 0.7 feet. Water level observations in the pumping well

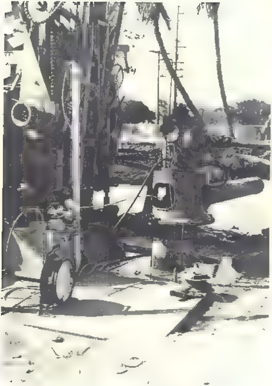


Fig. 28.--Pumping Test of Well 12, City of Leesburg

were made at the beginning of the test every five seconds. The natural time-drawdown graph of this first test, together with that of the second test, is shown on Figure 29.

Jaeger method computations for time-intervals starting at every 30 seconds are included in Table XVI. An analysis of this table indicated that the results obtained for the first time intervals up to 12:24-minutes do not resemble the artesian conditions present in the aquifer. Between the 12:24- and the 19.38-minute time intervals values obtained for both T and S vary between reasonable ranges, and from then on, S starts decreasing and T increasing in the typical recharge fashion. These variations can be observed better in Figure 30, which shows a plot of the values obtained for T and S for the different time intervals.

We have selected, therefore, those values corresponding to the sets of observations between the 12:24- and 19:38-minutes for the determination of the aquifer constants. The arithmetic mean of these values gives the following results:

$$T = 53{,}100 \text{ gpd/foot}$$

$$S = 0.00054$$

Figure 31 shows the semilog time-drawdown graph with the drawdown observations of this test. The Cooper and Jacob method with Lohman's modification gives for this pumping test the following results:

$$T = 50{,}300 \text{ gpd/foot}$$

$$S = 0.0012$$

The best fit we could get between the Theis-Cooper plot and the Cooper's Type Curves resulted in values of T, S and P' way out of line

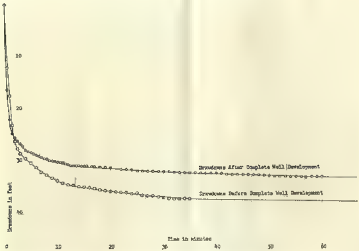


Fig. 29. Natural Time-Drawdown Graph, City of Leesburg, Well 12, Drawdowns in Pumping Well

TABLE XVI

CITY OF LEESBURG, FLORIDA
PUMPING TEST OF WELL 12
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
DRAWDOWN DATA FROM PUMPING WELL
TEST PERFORMED BEFORE COMPLETE DEVELOPMENT

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
0.5	1	12.1					
1.0	2	17.4	1.438	8.0×10^{-2}	2.03	16,300	1.1×10^{0}
1.0	1	17.4					
2.0	2	26.2	1.505	1.1×10^{-1}	1.74	9,300	1.7×10^{0}
1.5	1	23.2					
3.0	2	28.5	1.228	1.5×10^{-2}	3.64	17,900	6.7×10^{-1}
2.0	1	26.2					
4.0	2	29.4	1.122	9.8×10^{-4}	6.35	30,200	9.8×10^{-2}
2.5	1	27.6					
5.0	2	30.3	1.098	2.4×10^{-4}	7.76	35,800	3.5×10^{-2}
3.0	1	28.5					
6.0	2	31.2	1.093	1.7×10^{-4}	8.10	36,300	3.1×10^{-2}
4.0	1	29.4					
8.0	2	32.6	1.108	3.1×10^{-4}	7.50	32,200	6.6×10^{-2}
5.0	1	30.3					
10.0	2	33.7	1.112	5.4×10^{-4}	6.95	28,900	1.3×10^{-1}
6.0	1	31.2					
12.0	2	34.3	1.100	2.7×10^{-4}	7.64	31,200	8.3×10^{-2}
7.0	1	32.0					
14.0	2	34.8	1.086	1.1×10^{-4}	8.54	34,400	4.4×10^{-2}
8.0	1	32.6					
16.0	2	35.1	1.078	4.0×10^{-5}	9.55	38,100	2.0×10^{-2}
9.0	1	33.2					
18.0	2	35.4	1.087	9.0×10^{-6}	11.04	43,600	5.8×10^{-3}
10.0	1	33.7					
20.0	2	35.7	1.059	2.3×10^{-6}	12.41	48,600	1.8×10^{-3}
11.0	1	34.0					
22.0	2	35.9	1.056	1.2×10^{-6}	13.06	50,900	1.1×10^{-3}
12.0	1	34.3					
24.0	2	36.1	1.053	6.3×10^{-7}	13.70	53,100	6.6×10^{-4}
13.0	1	34.6					
26.0	2	36.4	1.051	3.8×10^{-7}	14.21	54,600	4.5×10^{-4}
14.0	1	34.8					
28.0	2	36.7	1.053	6.3×10^{-7}	13.70	52,200	7.6×10^{-4}

TABLE XVI - Continued

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
15.0	1	35.0					
30.0	2	36.9	1.053	6.3×10^{-7}	13.70	52,900	8.1×10^{-4}
16.0	1	35.1					
32.0	2	37.1	1.056	1.2×10^{-6}	13.06	49,200	1.6×10^{-4}
17.0	1	35.3					
34.0	2	37.1	1.051	3.8×10^{-7}	14.21	53,600	5.7×10^{-4}
18.0	1	35.4					
36.0	2	37.2	1.051	3.8×10^{-7}	14.21	53,500	6.1×10^{-4}
19.0	1	35.6					
38.0	2	37.3	1.048	1.6×10^{-7}	15.07	56,500	2.8×10^{-4}
20.0	1	35.7					
40.0	2	37.3	1.044	4.0×10^{-8}	16.46	61,600	8.1×10^{-5}
21.0	1	35.8					
42.0	2	37.3	1.043	2.7×10^{-8}	16.85	63,200	5.9×10^{-5}
22.0	1	35.9					
44.0	2	37.4	1.042	1.9×10^{-8}	17.20	64,300	4.4×10^{-5}
23.0	1	36.0					
46.0	2	37.4	1.039	5.4×10^{-9}	18.46	69,000	1.4×10^{-5}
24.0	1	36.1					
48.0	2	37.4	1.036	1.3×10^{-9}	19.88	74,400	3.8×10^{-6}
25.0	1	36.2					
50.0	2	37.4	1.033	2.1×10^{-10}	21.71	81,100	7.0×10^{-7}
26.0	1	36.4					
52.0	2	37.4	1.028	5.0×10^{-12}	25.44	95,200	2.0×10^{-8}
27.0	1	36.5					
54.0	2	37.5	1.028	5.0×10^{-12}	25.44	95,000	2.1×10^{-8}
28.0	1	36.6					
56.0	2	37.5	1.024	9.1×10^{-14}	29.45	110,000	4.6×10^{-10}
29.0	1	36.7					
58.0	2	37.5	1.022	6.4×10^{-15}	32.11	120,000	3.7×10^{-11}
30.0	1	36.9					
60.0	2	37.6	1.020	---	---	—	—

Q = 1220 gpm d = 16 inches r = 0.67 foot

$$T = \frac{114.6\ Q}{s} W(u) = \frac{114.6 \times 1220}{s} W(u) = 140{,}000 \frac{W(u)}{s}$$

$$S = \frac{u\ T\ t}{1.87\ r^2} = \frac{u\ T\ t'}{1.87 \times (0.67)^2 \times 1440} = \frac{u\ T\ t'}{1.21} \times 10^{-3}$$

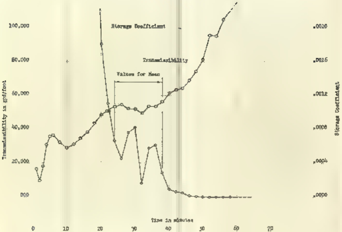


Fig. 36.—Values Obtained for T and S, City of Leesburg, Well 12, Before Complete Well Development

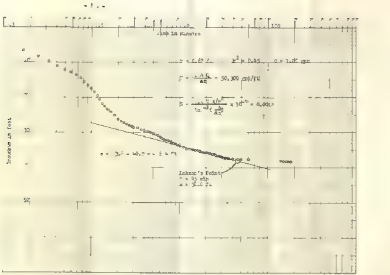


Fig. 31. Semilog. Time-Drawdown Graph, City of Leesburg, Well 1 Before Complete Well Development

$T = 9,300$ gpd

$S = 0.015$

$P' = 335,000$ gpd/sq.ft.!

Therefore, these results have been discarded.

The second drawdown test was performed in this well after the well was completely developed and the water practically free of sand and turbidity. Pumping was maintained at the constant rate of 1250 gpm and again the water level stabilized after approximately 160 minutes of pumping. However, this time the maximum drawdown was only 33.3 feet, instead of the 38.2 feet we had while pumping at 1220 gpm in the first test.

Figure 29 shows the natural time-drawdown graph with drawdowns from both the first and the second test, for the first 60 minutes of pumping. The difference in drawdowns for all corresponding time intervals is evident.

Jaeger method computations for the first 50 minutes of pumping, beginning with half-minute time intervals are included in Table XVII; and the values obtained for T and S in those computations are shown on Figure 32.

With the same reasoning used on the previous tests, we determined with the Jaeger method the following results for the aquifer constants

$T = 81,500$ gpd/foot

$S = 0.000014$

The semilog plot of the Cooper and Jacob method with the drawdown data of this second test is shown on Figure 33. With this method

TABLE XVIII

CITY OF LEESBURG, FLORIDA
PUMPING TEST OF WELL 12
JAEGER METHOD COMPUTATIONS FOR DIFFERENT TIME INTERVALS
DRAWDOWN DATA FROM PUMPING WELL
TEST PERFORMED AFTER COMPLETE DEVELOPMENT

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
0.5	1	16.3					
1.0	2	22.1	1.357	5.1×10^{-2}	2.45	15,900	6.7×10^{-1}
1.0	1	22.1					
2.0	2	25.5	1.153	3.2×10^{-3}	5.17	29,000	1.5×10^{-1}
1.5	1	24.7					
3.0	2	26.7	1.082	6.3×10^{-5}	9.10	48,700	7.6×10^{-3}
2.0	1	25.5					
4.0	2	27.7	1.086	9.2×10^{-5}	8.72	45,000	1.4×10^{-2}
2.5	1	26.2					
5.0	2	28.8	1.099	2.6×10^{-4}	7.68	38,100	4.1×10^{-2}
3.0	1	26.7					
6.0	2	28.8	1.078	4.1×10^{-5}	9.52	47,300	9.6×10^{-3}
3.5	1	27.2					
7.0	2	29.2	1.073	2.2×10^{-5}	10.15	49,600	6.4×10^{-3}
4.0	1	27.7					
8.0	2	29.6	1.068	1.1×10^{-5}	10.84	52,300	3.8×10^{-3}
4.5	1	28.1					
9.0	2	29.9	1.064	5.5×10^{-6}	11.59	55,100	2.5×10^{-3}
5.0	1	28.8					
10.0	2	30.1	1.045	5.8×10^{-8}	16.09	76,400	3.7×10^{-5}
5.5	1	28.6					
11.0	2	30.3	1.059	2.2×10^{-6}	12.45	58,700	1.2×10^{-3}
6.0	1	28.8					
12.0	2	30.5	1.059	2.2×10^{-6}	12.45	58,600	1.3×10^{-3}
6.5	1	29.0					
13.0	2	30.7	1.052	5.0×10^{-7}	13.93	64,900	3.5×10^{-4}
7.0	1	29.2					
14.0	2	30.8	1.055	1.0×10^{-6}	13.24	61,400	7.1×10^{-4}
7.5	1	29.4					
15.0	2	30.9	1.052	5.0×10^{-7}	13.93	64,400	4.0×10^{-4}
8.0	1	29.6					
16.0	2	30.9	1.043	2.7×10^{-8}	16.85	77,900	2.7×10^{-5}
8.5	1	29.7					
17.0	2	31.0	1.043	2.7×10^{-8}	16.85	77,700	2.9×10^{-5}
9.0	1	29.9					
18.0	2	31.1	1.040	8.3×10^{-9}	18.03	82,900	1.0×10^{-5}

TABLE XVII.— Continued

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	$W(u)$	T gpd/foot	S
9.5 19.0	1 2	30.0 31.2	1.040	6.3×10^{-9}	18.03	82,600	1.1×10^{-5}
10.0 20.0	1 2	30.1 31.3	1.040	6.3×10^{-9}	18.03	82,400	1.1×10^{-5}
10.5 21.0	1 2	30.2 31.4	1.040	8.3×10^{-9}	18.03	82,100	1.2×10^{-5}
11.0 22.0	1 2	30.3 31.6	1.042	1.2×10^{-8}	17.66	79,900	1.7×10^{-5}
11.5 23.0	1 2	30.4 31.6	1.039	5.4×10^{-9}	18.46	83,500	8.6×10^{-6}
12.0 24.0	1 2	30.5 31.7	1.039	5.4×10^{-9}	18.46	83,200	8.9×10^{-6}
12.5 25.0	1 2	30.6 31.8	1.039	5.4×10^{-9}	18.46	82,900	9.3×10^{-6}
13.0 26.0	1 2	30.7 31.8	1.036	1.3×10^{-9}	19.88	89,400	2.5×10^{-6}
13.5 27.0	1 2	30.7 31.8	1.036	1.3×10^{-9}	19.88	89,400	2.6×10^{-6}
14.0 28.0	1 2	30.8 31.9	1.036	1.3×10^{-9}	19.88	89,000	2.7×10^{-6}
14.5 29.0	1 2	30.8 31.9	1.036	1.3×10^{-9}	19.88	89,000	2.8×10^{-6}
15.0 30.0	1 2	30.9 32.0	1.036	1.3×10^{-9}	19.88	88,800	2.9×10^{-6}
15.5 31.0	1 2	30.9 32.0	1.036	1.3×10^{-9}	19.88	88,800	3.0×10^{-6}
16.0 32.0	1 2	30.9 32.1	1.039	5.4×10^{-9}	18.46	82,200	1.2×10^{-5}
16.5 33.0	1 2	30.9 32.1	1.039	5.4×10^{-9}	18.46	82,200	1.2×10^{-5}
17.0 34.0	1 2	31.0 32.1	1.036	1.3×10^{-9}	19.88	88,500	3.2×10^{-6}
17.5 35.0	1 2	31.1 32.2	1.035	7.6×10^{-10}	20.42	90,600	2.0×10^{-6}
18.0 36.0	1 2	31.1 32.2	1.035	7.6×10^{-10}	20.42	90,600	2.0×10^{-6}
18.5 37.0	1 2	31.2 32.2	1.032	1.1×10^{-10}	22.35	99,200	3.3×10^{-7}
19.0 38.0	1 2	31.2 32.2	1.032	1.1×10^{-10}	22.35	99,200	3.4×10^{-7}

TABLE XVII - Continued

t' min	$\frac{t'}{t'_o}$	s feet	$\frac{s}{s_o}$	u	W(u)	T gpd/foot	S
19.5	1	31.3	1.028	5.0×10^{-12}	25.44	113,000	1.8×10^{-8}
39.0	2	32.2					
20.0	1	31.9	1.032	1.1×10^{-10}	22.35	98,900	3.6×10^{-7}
40.0	2	32.3					
21.0	1	31.4	1.032	1.1×10^{-10}	22.35	98,600	3.8×10^{-7}
42.0	2	32.4					
22.0	1	31.6	1.028	5.0×10^{-12}	25.44	112,000	2.0×10^{-8}
44.0	2	32.5					
23.0	1	31.6	1.028	5.0×10^{-12}	25.44	112,000	2.1×10^{-8}
46.0	2	32.5					
24.0	1	31.7	1.028	5.0×10^{-12}	25.44	112,000	2.2×10^{-8}
48.0	2	32.6					
25.0	1	31.8	1.028	5.0×10^{-12}	25.44	111,000	2.3×10^{-8}
50.0	2	32.7					

Q = 1250 gpm d = 16 inches r = 0.67 foot

$$T = \frac{114.6\ Q}{1.87\ r^2} = \frac{u\ T\ t'}{1.87 \times (0.67)^2 \times 1440} = \frac{u\ T\ t'}{1.21} \times 10^{-3}$$

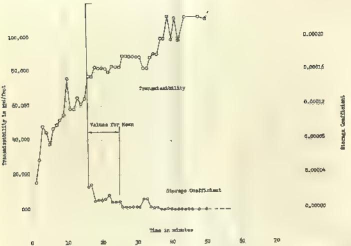


Fig. 32.-Values Obtained for T and S, City of Leesburg, Well 12, After Complete Well Development

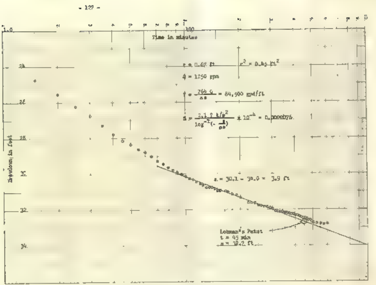


Fig. 33. Semilog Time-Drawdown Graph, City of Leesburg, Well 12, After Complete Development

we obtained for the transmissibility and the storage coefficient, the following results:

$$T = 84,500 \text{ gpd/foot}$$

$$S = 0.000007$$

Equilibrium conditions were reached in both tests at approximately 160 minutes after pumping began. Therefore, with the maximum drawdowns for each test, with Jacob's equation (20) and with the same procedure we used in Chapter X we have determined P' for the values obtained with each method in each test. All these values are summarized in Table XVIII.

An analysis of Table XVIII clearly shows the importance of having the well completely developed before trying to determine aquifer constants with these methods. It also indicates a good agreement between the results obtained with the Jaeger method and the semilog plot of Cooper and Jacob with Lohman's modification.

TABLE XVIII

CITY OF LEESBURG, FLORIDA
PUMPING TEST OF WELL 12
SUMMARY OF RESULTS OBTAINED FOR AQUIFER CONSTANTS USING DRAWDOWN DATA FROM THE PUMPING WELL

Remarks	Method	Transmissibility T gpd/foot	Storage Coefficient S –	Vertical Permeability P' gpd/sq.ft.
Test Before Complete Well Development	Jaeger	53,100	0.00054	13.6
	Cooper & Jacob	50,300	0.0012	38.2
Test After Complete Well Development	Jaeger	81,500	0.000014	0.24
	Cooper & Jacob	84,500	0.000007	0.13

XII. SUMMARY AND CONCLUSIONS

Three methods of determining aquifer constants with drawdown data from satellite wells have been used in the determination of the same aquifer constants with drawdown data from the pumping wells. The three methods are the Jaeger method, the Cooper and Jacob method with Lohman's modification; and the Theis method, with Cooper's Type Curves modification whenever necessary or possible.

Drawdown data from four pumping tests have been used in this evaluation. Experience from the application of those methods to data from the first two pumping tests indicated the necessity of making fast and accurate observations in the pumping well immediately after pumping is started. A nitrogen-bubbler system was developed for such a purpose.

The actual radius of the well has been used to substitute for r, the distance from the center of the well to the point of observation, in each and all mathematical expressions employed with the different methods and with drawdown data from the pumping well.

The results obtained from the different tests with the various methods employed are summarized in Table XIX.

Equilibrium conditions were reached in the last tests because of leaky-aquifer effects. With the values of the maximum stabilized drawdowns and Jacob's steady-state equation (equation 20 of Chapter VII) we have calculated the values of the vertical permeability P' for the values of T and S calculated with the different methods. These results

TABLE XIX

SUMMARY OF RESULTS FOR T AND S

Pumping Test	Type of		Flow Rate gpm	Well for Drawdown Measurement	Method	T gpd/ft	S -
	Well	Aquifer					
Marianao Well 13	Open-hole	Water-table to Semi-artesian	4980	Pumping	Jaeger	2,430,000	0.009
					Cooper-Jacob	2,340,000	0.014
				Satellite	Jaeger	1,920,000	0.083
					Cooper-Jacob	2,080,000	0.071
					Theis	2,000,000	0.081
Cervecera Antillas Well 2	Open-hole	Water-table to semi-artesian	1280	Pumping	Jaeger	136,000	0.026
					Cooper-Jacob	136,000	0.011
				Satellite	Jaeger	111,000	0.023
					Cooper-Jacob	134,000	0.018
					Theis	115,000	0.018
Fort Pierce Well T 10A	Screened	Low-artesian	400	Pumping	Jaeger	20,300	0.0032
					Cooper-Jacob	21,100	0.0025
					Theis-Cooper	5,300	0.34
				Satellite	Jaeger	28,800	0.0013
					Cooper-Jacob	45,000	0.0009
					Theis-Cooper	32,300	0.0014

TABLE XIX - Continued

	Type of		Flow Rate gpm	Well for Drawdown Measuremen.	Method	T gpd/ft	S -
	Well	Aquifer					
Leesburg Well 12	Open-hole	Artesian	1220[1]	Pumping	Jaeger	53,100	0.00054
					Cooper-Jacob	50,300	0.0012
			1250[2]	Pumping	Jaeger	81,500	0.000015
					Cooper-Jacob	84,500	0.000007

Notes: [1] Pumping test performed before complete well development.

[2] Pumping test performed after complete well development.

for the vertical permeability P' are summarized in Table XX together with those obtained from the Type Curves (Theis-Cooper method).

It has been pointed out that aquifer constants cannot be regarded as exact mathematical figures, but rather, as coefficients which express the water-bearing characteristics of an aquifer and also which may permit us to estimate the drawdowns produced by well pumping in the pumping well itself and in its vicinities.

With the above considerations in mind and the results obtained we are led to the following conclusions:

1. For the determination of aquifer constants using water-level observations in the pumping well it is necessary to make accurate and numerous observations, at least every 30 seconds, immediately after pumping is started.

2. The results obtained for aquifer constants with drawdown data from the pumping wells, with the Jaeger and the Cooper and Jacob methods, seem to agree, within reasonable limits, with those obtained from water-level observations in satellite wells.

3. The statement in paragraph 2 above, is especially true in open-hole wells, where in the opinion of this writer, drawdown data from the pumping well identify more accurately aquifer conditions and give a better picture of any field deviation from the theoretical assumptions.

4. In the instance of the screened well, results for aquifer constants determined with drawdown data from the pumping well gave slightly smaller values for T and slightly greater values for S than those determined from the satellite well. However, the results with data from the pumping well defined more accurately the drawdown

TABLE XX

SUMMARY OF RESULTS FOR THE VERTICAL PERMEABILITY $\underline{P}'$ UNDER LEAKY-AQUIFER and STEADY-STATE CONDITIONS

Pumping Test	Well for Drawdown Measurement	Method of Determining T and S	P' gpd/sq.ft.
Fort Pierce Well T 10A	Pumping	Jaeger	28
		Cooper & Jacob	20
		Theis & Cooper	0.06
	Satellite	Jaeger	30
		Cooper & Jacob	5
		Theis & Cooper	26
Leesburg Well 12	Pumping (1)	Jaeger	13.6
		Cooper & Jacob	38.2
	Pumping (2)	Jaeger	0.24
		Cooper & Jacob	0.13

Notes: (1) Pumping test performed before complete well development.

(2) Pumping test performed after complete well development.

conditions in the pumping well itself and were on the safe side for drawdown forecasts in the vicinities of that well.

5. It is extremely important to conduct the drawdown test in a pumping well only after it has been completely developed.

6. The writer has not been successful with recovery data from the pumping well. These data seem to be affected by the backflow towards the well of all the water in the pump and in the pump column.

This work and the previous conclusions must not lead us to the belief that observations in satellite wells will be no longer necessary sometime in a near future. Much more research is needed in this field, using data from the pumping well and from as many satellite wells as possible. In the interim, however, the methods used in this work may offer a tool for determining aquifer constants where no observation wells are available. This is the case, unfortunately, that the groundwater engineer finds most often in the field.

Future research should include considerations using multiple-step drawdown tests as developed by Jacob with modifications by Rorabaugh.

Finally it is significant to point out that computations with the Jaeger method are most adaptable to be used with digital computers, which will facilitate considerably the desk work involved. The Jaeger method seems to be the most valuable tool in these evaluations.

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The writer was born in Havana, Cuba, on June 1, 1927; attended the Colegio de Belen (1934-1945) where he got his degree of Bachiller en Ciencias. In 1945 he entered the University of Havana where he graduated as a Civil Engineer in 1950. In June, 1959, he entered the University of Florida and received his Master of Science in Engineering degree in June, 1960.

From 1950 to 1954 he worked with Comision de Fomento Nacional (Cuba). During his work with CFN he was in charge of the design of 15 waterworks in the country and also of the investigations of the water resources for the City of Havana. As a result of these investigations a new 100 million gallons per day well field was developed for that city, with Black and Associates, Inc., as consulting engineers.

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This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Engineering and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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